Geometry, Quarter 2, Unit 2.1
Proving Theorems Involving Parallelograms
and Using Coordinates to Prove Simple
Geometric Theorems

Overview

Number of instructional days: 15 (1 day = 45 minutes)

Content to be learned

• Identify different parallelograms, which include rhombuses, rectangles, and squares.
• Prove the following about parallelograms:
  o Opposite sides are congruent.
  o Opposite angles are congruent.
  o Diagonals of a parallelogram bisect each other.
• Prove all properties of parallelograms, including rectangles, rhombuses, and squares.
• Apply the distance formula, slope of a line, midpoint formula, and Pythagorean Theorem to classify quadrilaterals on the coordinate plane.
• Compute perimeters of polygons and areas of triangles and rectangles on the coordinate plane.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

• Analyze given information to prove if a quadrilateral is a parallelogram, rectangle, rhombus, or square.
• Make a conjecture and identify and execute appropriate strategy to classify a quadrilateral on the coordinate plane.
• Explain correspondences between formulas and important features of polygons.

Construct viable arguments and critique the reasoning of others.

• Use prior knowledge of congruence in constructing arguments for parallelograms.
• Communicate conclusions using a variety of ways of writing proofs for parallelograms.
• Determine geometric properties by reasoning inductively from a given set of coordinates.

Essential questions

• How can you prove that a quadrilateral is a parallelogram, rectangle, rhombus, or square?
• How can you use coordinate geometry to identify quadrilaterals?
• How can you find the perimeter of a polygon on the coordinate plane?
• How can you find the area of triangles and rectangles on the coordinate plane?
Written Curriculum

Common Core State Standards for Mathematical Content

<table>
<thead>
<tr>
<th>Congruence</th>
<th>G-CO</th>
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<tbody>
<tr>
<td><strong>Prove geometric theorems</strong> [Focus on validity of underlying reasoning while using variety of ways of writing proofs]</td>
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<tr>
<td>G-CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.</td>
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<tr>
<th>Expressing Geometric Properties with Equations</th>
<th>G-GPE</th>
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<tbody>
<tr>
<td><strong>Use coordinates to prove simple geometric theorems algebraically</strong> [Include distance formula; relate to Pythagorean theorem]</td>
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<tr>
<td>G-GPE.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point ((1, \sqrt{3})) lies on the circle centered at the origin and containing the point ((0, 2)).</td>
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<tr>
<td>G-GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.</td>
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Common Core Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

3. **Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them...
into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.

Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

**Clarifying the Standards**

**Prior Learning**

In grade 7, students found the length and area of two-dimensional figures. (7.G.6)

In Unit 1.2, students used the slope formula to show that lines are parallel, perpendicular, or neither. They applied the distance formula to determine the length of a segment.

**Current Learning**

Students prove that a quadrilateral is a parallelogram using triangle congruence, and they verify with coordinate geometry. Students define the different types of parallelograms, which include a rectangle, rhombus, and square. They use properties of parallelograms to classify quadrilaterals. Students learn how to measure the perimeter of polygons and areas of triangles and rectangles using coordinate geometry.

**Future Learning**

Students will apply the properties of parallelograms throughout this course under the concepts of similarity, trigonometry, and three-dimensional figures.

**Additional Findings**

None at this time.

**Writing Team Notes**

This unit of study exposes student to proofs that rely on algebra and the coordinate plane.

A good understanding of plotting points, using the distance, slope, midpoint, perimeter, and area formulas is essential for this unit. Accuracy is essential when making computations.

A common misconception is students will get a negative number under the square root when applying the distance formula. Remind students to use order of operations correctly.
Number of instructional days: 13  (1 day = 45 minutes)

Content to be learned

• Construct and discover the properties of dilations where the center is a given point (not always the origin or the center of the figure).
• Verify that the line segment connecting the center to the image and the line segment connecting the center to the preimage is on the same line and the ratio of the two segments is equal to the scale factor.
• Using the definition that similarity is a series of transformations, determine if two figures are similar.
• Explain the meaning of similarity for triangles by showing corresponding angles are congruent and corresponding sides are in proportion.
• Through transformations, establish the AA criterion for proving triangles are similar.
• Prove the following theorems about triangles:
  o A line parallel to one side of a triangle divides the other two sides of the triangle proportionally and conversely (Triangle Proportionality Theorem).
  o Pythagorean Theorem—using similar triangles (i.e., geometric mean).
• Apply proportional reasoning coordinate geometry to find unknown measures of geometric figures and solve real-world design problems.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.
• Understand and use prior learning of transformations in constructing arguments to prove polygons are similar.
• Justify conclusions, communicate them to others, and respond to the arguments of others regarding triangle similarity.
• Distinguish between correct and flawed logic and explain why.

Model with mathematics.
• Read, understand, and draw conclusions from a dilation.
• Verify similarity of polygons through transformations.
• Analyze similar triangles to draw conclusions on corresponding parts.

Essential questions

• What are the properties of a dilation?
• How are polygons similar?
• How can transformations be used to establish criterion for similarity?
• How are similar triangles used in solving problems in everyday life?
### Written Curriculum

**Common Core State Standards for Mathematical Content**

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<tr>
<th>Similarity, Right Triangles, and Trigonometry</th>
<th>G-SRT</th>
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<tr>
<td><strong>Understand similarity in terms of similarity transformations</strong></td>
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<tr>
<td>G-SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor:</td>
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<tr>
<td>a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.</td>
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<td>b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.</td>
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<tr>
<td>G-SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.</td>
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<tr>
<td>G-SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.</td>
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<tr>
<td><strong>Prove theorems involving similarity</strong></td>
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<tr>
<td>G-SRT.4 Prove theorems about triangles. <em>Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.</em></td>
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<tr>
<td>G-SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.</td>
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<tr>
<th>Modeling with Geometry</th>
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<tr>
<td><strong>Apply geometric concepts in modeling situations</strong></td>
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<tr>
<td>G-MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*</td>
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**Common Core Standards for Mathematical Practice**

3. **Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings,
diagrams, and actions. Such arguments can make sense and be correct, even though they are not
generalized or made formal until later grades. Later, students learn to determine domains to which an
argument applies. Students at all grades can listen or read the arguments of others, decide whether they
make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in
everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition
equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a
school event or analyze a problem in the community. By high school, a student might use geometry to
solve a design problem or use a function to describe how one quantity of interest depends on another.
Mathematically proficient students who can apply what they know are comfortable making assumptions
and approximations to simplify a complicated situation, realizing that these may need revision later. They
are able to identify important quantities in a practical situation and map their relationships using such
tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships
mathematically to draw conclusions. They routinely interpret their mathematical results in the context of
the situation and reflect on whether the results make sense, possibly improving the model if it has not
served its purpose.

Clarifying the Standards

Prior Learning

In grade 8, students described the effect of dilations, translations, rotations, and reflections on two-
dimensional figures using coordinates. (8.G.3) Students understood that a two-dimensional figure is
similar to another if the second can be obtained from the first by a sequence of transformations and vice
versa. (8.G.4)

Current Learning

Students develop a more in-depth understanding of dilations with the center point at various locations.
They develop an understanding of similarity within polygons. Students reinforce reasoning and
constructing logical arguments through writing proofs. They prove theorems about triangles, including
the AA criterion, Pythagorean Theorem, and Triangle Proportionality Theorem.

Future Learning

Similarity and use of the Pythagorean Theorem will provide a foundation for further study of
trigonometry within this course (Unit 2.3), Algebra 2 (Units 3.2, 3.3, and 4.1), and Precalculus.

Additional Findings

The following websites can be helpful:

- www.math-drills.com/geometry.shtml (worksheets on dilations [and other transformations])
- www.regentsprep.org/Regents/math/geometry/GT3/indexGT3.htm (lesson for dilations)
Geometry, Quarter 2, Unit 2.3
Solving Problems Using Trigonometry

Overview

Number of instructional days: 7  (1 day = 45 minutes)

Content to be learned

• Define trigonometric ratios of acute angles within a right triangle.
• Understand and apply the relationship between the sine and cosine of complementary angles.
• Solve applied problems involving right triangles using trigonometric ratios or the Pythagorean Theorem.

Standards with a (+) symbol are additional mathematics that students should learn to take advance courses such as Calculus, Advanced Statistics, or Discrete Mathematics.

• Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle. ($A = \frac{1}{2}$ the product of two sides and the sine of the included angle.)
• Prove the Laws of Sines and Cosines and use them to solve problems.
• Understand and apply the Laws of Sines and Cosines to find unknown measurements in right and oblique triangles (e.g., surveying problems, aviation problems).

Mathematical practices to be integrated

Model with mathematics.

• Apply trigonometric ratios and the Pythagorean Theorem to solve problems in everyday life.
• Analyze relationships between the angles and sides of a right triangle and oblique triangle.

Attend to precision.

• Accurately label units of measure.
• Calculate accurately and efficiently.
• Express numerical answers with a degree of precision appropriate for the problem context.

Essential questions

• What are the trigonometric ratios for a right triangle?
• What are the different ways to solve right triangles?

• (+) How do you find the area of an oblique triangle?
• (+) How do you prove and use the Laws of Sines and Cosines?
Written Curriculum

Common Core State Standards for Mathematical Content

**Similarity, Right Triangles, and Trigonometry**

| **G-SRT.6** | Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. |
| **G-SRT.7** | Explain and use the relationship between the sine and cosine of complementary angles. |
| **G-SRT.8** | Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.* |

**Apply trigonometry to general triangles**

| **G-SRT.9** | (+) Derive the formula $A = 1/2 ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. |
| **G-SRT.10** | (+) Prove the Laws of Sines and Cosines and use them to solve problems. |
| **G-SRT.11** | (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). |

**Common Core Standards for Mathematical Practice**

4. **Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6. **Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
Clarifying the Standards

Prior Learning

In Unit 2.2, students derived and proved the Pythagorean Theorem using similar triangles.

Current Learning

Students learn the basic trigonometric ratios for a right triangle. They develop an understanding of how to apply trigonometric ratios and the Pythagorean Theorem to solve right triangles.

(+) Students derive the formula $A = \frac{1}{2} ab \sin(C)$ for area of an oblique triangle.

(+) Students prove, understand, and apply the Laws of Sines and Cosines to solve oblique triangles.

Future Learning

In Algebra 2, students will define trigonometric functions and prove and apply the Pythagorean identity (Units 3.2 and 3.3).

(+) In Precalculus, students will graph trigonometric functions, prove trigonometric identities, and construct inverse trigonometric functions.

Additional Findings

The following website can be helpful:

- www.regentsprep.org/Regents/math/algtrig/math-algtrig.htm#m7