Grade 3 Mathematics, Quarter 1, Unit 1.1

Number System—Addition and Subtraction, Place Value and Patterns in Addition

Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned

- Solve two-step word problems using addition and subtraction.
- Represent problems using equations where a letter stands for an unknown quantity.
- Identify arithmetic patterns in addition and subtraction.
- Round whole numbers to 10s and 100s using place value.
- Add and subtract within 1,000 using strategies and algorithms based on place value and properties of operations.
- Use the relationship between addition and subtraction to solve problems.
- Check the reasonability of solutions using strategies such as mental computation, estimation, and rounding.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Read and understand word problems.
- Plan a strategy to solve problems.
- Assess solutions for reasonableness and change strategies if necessary.

Construct viable arguments and critique the reasoning of others.

- Justify answers and explain thinking.
- Look at other students' solutions for additional strategies.

Model with mathematics.

- Make connections to everyday life when solving problems.
- Make connections to previously learned strategies.

Essential questions

- How do you explain the relationship between addition and subtraction?
- How did you solve the two-step word problem? Explain your thinking.
- How can you use an equation to represent the problem and the unknown?
- How can you solve the problem? Explain your thinking.
- What patterns did you notice when using addition and subtraction? How did you find them?
- How do you use place value to round numbers?
- How can the use of estimation make logical problem-solving decisions?
- How can you check your answer?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking 3.OA

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8 Solve two-step word problems using ~~the four~~ operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3]

[3] This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

3.OA.9 Identify arithmetic patterns (including patterns in the addition table ~~or multiplication~~ table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Number and Operations in Base Ten 3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.[4]

[4] A range of algorithms may be used.

3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

3.NBT.2 ~~Fluently~~ add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In grade 2, students solved one- and two-step word problems involving addition and subtraction within 100. They added and subtracted within 100 using a symbol for an unknown number. Students skip counted by 2s, 5s, 10s, and 100s. They also added up to four digits. Students added and subtracted within 1,000. They read and wrote numbers to 1,000. Students gained a basic knowledge of place value. Rounding was not previously learned.

Current Learning

In grade 3, students solve two-step addition and subtraction word problems using mental computation, estimation, and rounding to the nearest 10 and 100. They use a letter to represent an unknown quantity. Students identify addition patterns while adding and subtracting within 1,000.

Later in the year, students become fluent using addition and subtraction within 1,000 using four operations and two-step word problems. They continue rounding multidigit numbers to the nearest 10s and 100s.

Future Learning

In grade 4, students will continue to seek patterns following a given rule and explain the rule. They will solve multistep word problems using all four operations. Students will be expected to use standard algorithms.

Additional Findings

According to *Curriculum Focal Points*, the expectation for third graders is to "develop fluency in adding, subtracting, multiplying, and dividing whole numbers." (p. 30)

According to *Principles and Standards for School Mathematics*, third graders

- "… develop and use strategies to estimate the results of whole number computation and to judge the reasonableness of such results."
- "… develop fluency in adding subtracting, multiplying, and dividing whole numbers." (p. 392)

Grade 3 Mathematics, Quarter 1, Unit 1.2

Introduction to Fractions and Partitioning Shapes into Parts with Equal Areas

Overview

Number of instructional days: 5 (1 day = 45 minutes)

Content to be learned

- Understand a fraction as an equal part of a whole.
- Use fractions to decompose and compose a whole.
- Explore partitioning a shape into parts with equal area.
- Describe the area of each part as a unit fraction of the whole.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of fractional quantities and their relationships in problem situations.
- Decompose and compose a whole using fractional parts.
- Recognize the unit fraction as the building blocks of fractions.

Construct viable arguments and critique the reasoning of others.

- Justify answers and explain thinking.
- Look at other students' solutions for additional strategies.

Essential questions

- What ways can you partition a shape into parts with equal area and label them?
- How do you use concrete materials and drawings to show understanding of fractions?
- When you are given a fraction of a shape, how do you complete the whole?
- How can you describe a fraction?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions[5]	**3.NF**

[5] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

Develop understanding of fractions as numbers.

3.NF.1 Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

Geometry	**3.G**

Reason with shapes and their attributes.

3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Clarifying the Standards

Prior Learning

In grade 2, students have partitioned circles and rectangles into halves, thirds, fourths, etc. with an understanding of the words *half of, a third of*, etc. Students partitioned a rectangle into rows and columns having same-sized squares and counted to find the total.

Current Learning

In grade 3, students develop an understanding of fractions, beginning with unit fractions. They decompose and compose a whole into fractional parts having equal area. Students need to understand that the whole is specific and the meaning of "equal parts."

Later in the quarter, students represent fractions on a number line. They understand equivalent fractions comparing size and shape. Students compare fractions having the same numerator or denominator.

Future Learning

In grade 4, students will develop an understanding of fraction equivalence. They will compare fractions with different numerators and denominators. Students will add and subtract fractions as well as multiply fractions by whole numbers.

Additional Findings

According to *Curriculum Focal Points*, "Students develop an understanding of the meanings and uses of fractions to represent parts of a whole and parts of a set." In addition, "Students strengthen their understanding of fractions as they confront problems in linear measurement that call for more precision than the whole unit allowed them in grade 2 ..." (p. 15)

According to *Principles and Standards for School Mathematics,* "Students learn how fractions are related to each other and the unit whole, and how they are represented. Students gain facility in comparing fractions. They build their understanding of fractions as parts of a whole and as division." (pp. 149–150)

Grade 3 Mathematics, Quarter 1, Unit 1.3

Fractions on a Number Line and a Line Plot

Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned

- Recognize that a whole partitioned in equal parts is composed of unit fractions of equal size.
- Represent fraction *a/b* on a number line from 0 to 1 by partitioning unit fractions.
- Use a ruler to measure using whole numbers, 1/2 and 1/4 of an inch.
- Make a line plot with horizontal scale marked off as whole numbers, 1/2, and 1/4.

Mathematical practices to be integrated

Use appropriate tools strategically.

- Identify tools that are relevant and helpful.
- Use technology tools to deepen understanding of math concepts.
- Use tools to create or solve problems.
- Use tools to visualize possible solutions so errors can be detected.

Attend to precision.

- Demonstrate precise use of math terms, symbols, and units of measure.
- Calculate accurately and efficiently.
- Formulate clear explanations and communicate reasoning.

Essential questions

- How do you represent a fraction on a number line you create?
- How do you explain where to place a fraction on a number line?
- How can you justify where your fraction belongs on a number line?
- How can you create a line plot to show your data, including fractions?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions[5]	**3.NF**

[5] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

Develop understanding of fractions as numbers.

3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.

b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

Measurement and Data	**3.MD**

Represent and interpret data.

3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

Common Core Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 2, students created line plots where the scale was marked off in whole number units. They partitioned circles and rectangles into two, three, or four equal shares using the words *halves*, *thirds*, *quarters*, *half of*, *a third of*, and *a fourth of*. Students described the whole as two halves, three thirds, and four fourths. They represented whole number as lengths from zero on a number line diagram with equally spaced points.

Current Learning

In grade 3, students learn to understand fractions as a number on the number line and represent fractions on a number line diagram. Unit fractions are fractions with one as the numerator. Third graders are expected to recognize and locate fractions with denominators of 2, 3, 4, 6, and 8. Students use partitioning and iterations to model these fractions. They measure lengths with rulers marked with halves and fourths. Students collect fractional data and place it on a line plot. Later in the quarter, they develop an understanding of equivalent fractions.

Future Learning

In grade 4, students will explain why a fraction *a/b* is equivalent by using a visual fraction model, with attention to how the number and size parts differ even though two fractions are the same size (e.g., 1/2 = 2/4). Students will solve problems involving addition and subtraction of fractions using information presented in line plots. They will include 1/8 in measurement problems.

Additional Findings

According to *Principles and Standards for School Mathematics*, students should "represent data using tables and graphs such as line plots, bar graphs and line graphs." (p. 400)

According to *PARCC Model Content Frameworks for Mathematics* (October 2011), "It is critical that students at this grade are able to place fractions on a number line diagram and understand them as a related component of their ever-expanding number system." (p. 16) This is a major cluster of work for this grade level.

According to *Common Core Progressions K–3, Categorical Data*, "The students can draw a segment of a number line diagram that includes fractional measurement values with tick marks including specific values on the measurement scale." (p. 10)

Grade 3 Mathematics, Quarter 1, Unit 1.4

Introduction to Multiplication—Products of Whole Numbers and Area Related to Mulitpication and Addition

Overview

Number of instructional days: 15 (1 day = 45 minutes)

Content to be learned

- Recognize the total as the number of objects in a number of groups.
- Solve multiplication word problems involving equal groups and arrays.
- Relate the area of a rectangle to multiplying the side lengths.
- Represent the Distributive Property of Multiplication using arrays.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities and relationships in problem situations.
- Break down a word problem into components for better understanding.
- Translate a word problem into a number sentence to accurately solve.

Look for and make use of structure.

- Relate unknown facts to simpler known facts.
- Use area models to represent the properties of multiplication.
- Look for patterns and repetitions when solving problems.

Look for and express regularity in repeated reasoning.

- Look for general methods to solve problems.
- Find shortcuts for calculations.
- Evaluate intermediate results for reasonableness.

Essential questions

- What is multiplication?
- How do you use multiplication to determine the products of whole numbers?
- How do you use multiplication to solve word problems?
- How do you find the area of a rectangle using multiplication/addition?
- How can you use arrays to solve for area?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking 3.OA

Represent and solve problems involving multiplication and division.

3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*

3.OA.3 Use multiplication ~~and division within 100~~ to solve word problems in situations involving equal groups, arrays, ~~and measurement quantities~~, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

[1] See Glossary, Table 2.

Measurement and Data 3.MD

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.7 Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In

the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In grade 2, students progressed from using addition to multiplication by finding the total number of objects in arrays up to 5×5. They counted objects by twos using skip counting and repeated addition.

Current Learning

Multiplication is the major focus for grade 3. Students interpret products as whole numbers. They use multiplication to solve word problems involving equal groups, arrays, and drawings with equations of unknown symbols. Students find the area of a rectangle using multiplication for side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. They use arrays to show the Distributive Property of Multiplication.

Future Learning

In grade 4, students will develop fluency with efficient procedures for multiplying whole numbers. They will need to know that multiplication is a comparison—multiplicative comparison. Students will multiply and divide to solve word problems with an unknown symbol and be able to distinguish multiplicative comparisons from additive comparisons. Students will solve multistep word problems using the four operations. They will find all factor pairs and recognize whole number multiples. Students will use area formulas.

Additional Findings

According to *PARCC Model Content Framework for Mathematics,* "Word problems involving equal groups equal groups, arrays and measurement quantities can be used to build students' understanding of a skill with multiplication and division, as well as to allow students to demonstrate their understanding of and skill with these operations." (p. 16)

According to *Curriculum Focal Points*, "Students understand the meaning of multiplication and division of whole numbers through the use of representations (e.g., area models)." (p. 15)