Grade 5 Mathematics, Quarter 2, Unit 2.1

Adding and Subtracting Fractions

Overview

Number of instructional days: 14 (1 day = 45 minutes)

Content to be learned

- Use equivalent fractions to add and subtract fractions with unlike denominators, including mixed numbers.
- Solve word problems involving addition and subtraction of fractions.
- Use visual fraction models and equations to add and subtract when solving word problems.
- Use benchmark fractions and number sense of fractions to estimate and check reasonableness of sums and differences.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Attend to the meaning of quantities, not just how to compute them.
- Know and flexibly use different properties of operations.

Construct viable arguments and critique the reasoning of others.

- Analyze situations by breaking them into cases.
- Recognize and use counterexamples.

Model with mathematics.

- Solve problems arising in everyday life.
- Routinely interpret their mathematical results and reflect on whether the results make sense.

Essential questions

- How is computation using fractional numbers similar and different from computation using whole numbers?
- What real-world example can you give where you add or subtract fractions to solve a problem?
- How can you use visual representations for adding and subtracting fractions?
- How do you add and subtract fractions?
- How are benchmarks fractions used to estimate sums and differences?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions	5.NF

Use equivalent fractions as a strategy to add and subtract fractions.

5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

Students understood addition and subtraction of fractions as joining and separating parts referring to the same whole and decomposed fractions. They added and subtracted mixed numbers. Students solved word problems involving addition and subtraction of fractions using visual fraction models and equations to represent models.

Current Learning

With a focus on equivalence, students add and subtract fractions with unlike denominators. They make reasonable estimates with fraction sums and differences (*Curriculum Focal Points,* p. 17). Students compute with fractions to solve problems, including measurement problems.

Future Learning

According to *Curriculum Focal Points*, sixth graders will apply and extend their previous understanding of fractions, multiplication, and division to explain how to multiply and divide fractions. (p. 18) They will use the multiplication and division of fractions to solve problems, including in depth multistep and measurement problems.

Additional Findings

According to progressions for the Common Core State Standards, it is not necessary to find a least common denominator to calculate fraction sums. The effort of finding a least common dominator is a distraction from understanding algorithms for adding fractions.

Grade 5 Mathematics, Quarter 2, Unit 2.2

Understanding Multiplication of Fractions

Overview

Number of instructional days: 14 (1 day = 45 minutes)

Content to be learned

- Explain why multiplying a given number by a fraction can result in a number either greater than or less than the given number.
- Interpret multiplication as scaling by comparing the size of the product to the size of its factors.
- Use visual fraction models and equations to solve real-world problems involving multiplication of fractions and mixed numbers.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Analyze situations by breaking them into cases.
- Construct arguments using concrete referents such as objects, drawings, diagrams, and actions.
- Listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Model with mathematics.

- Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
- Make assumptions and approximations to simplify a complicated situation.

Use appropriate tools strategically.

- Consider the available tools when solving a mathematical problem.
- Use pencil and paper and concrete models.

Essential questions

- What happens when you multiply a fraction greater than one by a number greater than one? Why?
- What happens when you multiply a fraction less than one by a number less than one? Why?
- How do you determine when to multiply fractions when solving real-world problems?
- Using visual fraction models or equations, how can you represent a multiplication problem?
- How can multiplication be interpreted as scaling?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions	5.NF

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.5 Interpret multiplication as scaling (resizing), by:

a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They

are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

In grade 4, students multiplied a fraction by a whole number and solved word problems involving multiplication of a fraction by a whole number using fraction models and equations to represent the problem. They explained why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Students used this principle to recognize and generate equivalent fractions. Fourth graders compared two fractions with different numerators and different denominators by comparing to a benchmark fraction such as 1/2.

Current Learning

Multiplication as "scaling" is an important concept for students to understand. Previous work with multiplication by whole numbers enables students to see multiplication by numbers bigger than one as producing a larger quantity (e.g., when a recipe is doubled). Fifth graders work with multiplying by unit fractions and interpreting fractions in terms of division. This helps them to see that multiplying a quantity by a number less than one produces a product less than one (e.g., when the budget of a large state university is multiplied by 1/2).

In grade 5, students apply and extend their previous understanding of multiplication to multiply a fraction or whole number by a fraction. They also learn the relationship between fractions and division, allowing them to divide any whole number by another whole number except zero and express the answer in the form of a fraction or mixed number. Students apply and extend their previous understanding of multiplication and division to divide a unit fraction by a whole number or a whole number by a unit fraction.

Future Learning

In grade 6, students will interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions. They begin to use their understanding of fractions in the study of ratios, proportional relationships, and unit rates.

Additional Findings

Students need to develop strategies for judging the relative sizes of numbers. They should understand more deeply the multiplicative nature of the number system, including the structure of 786 as 7 times 100 plus 8 times 10 plus 6 times 1.

With models or calculators, students can explore dividing by numbers between zero and one (e.g., 1/2) and find that the quotient is larger than the original number. Explorations such as these help dispel common, but incorrect, generalizations such as "Division always makes things smaller."

Grade 5 Mathematics, Quarter 2, Unit 2.3

Multiplying and Dividing Fractions

Overview

Number of instructional days: 15 (1 day = 45 minutes)

Content to be learned

- Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
- Find the area of a rectangle with fractional side lengths by tiling with unit squares of appropriate unit fractions.
- Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
- Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$).
- Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers.
- Fluently multiply multidigit whole numbers using the standard algorithm.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Start by explaining to themselves the meaning of a problem and looking for entry points to its solution.
- Explain correspondences between equations.
- Check answers to problems using a different method, and continually ask, "Does this make sense?"

Reason abstractly and quantitatively.

- Bring two complementary abilities to bear on problems involving quantitative relationships: *decontextualize* and *contextualize.*
- Create a coherent representation of the problem at hand, attending to the meaning of quantities and knowing and flexibly using different properties of operations and objects.

Look for and express regularity in repeated reasoning.

- Notice if calculations are repeated, and look for both general methods and shortcuts.
- Maintain oversight of the process, while attending to the details.

Essential questions

- What is the connection between a fraction and division?
- How do you multiply a fraction by a whole number?
- What is the process for dividing unit fractions by whole numbers and whole numbers by unit fractions?
- How do you multiply a fraction by a fraction?
- How do you find the area of a rectangle with fractional sides?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions	5.NF

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)*

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5.NF.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

Number and Operations in Base Ten	5.NBT

Perform operations with multi-digit whole numbers ~~and with decimals to hundredths~~.

5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using

concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In grade 4, students learned the properties of equivalent fractions. They also compared, added, and subtracted fractions. Students added fractions with different denominators where one denominator is a divisor of the other, so that only one fraction has to be changed. They made connections with adding and multiplying fractions (e.g., $2/3 = 1/3 + 1/3 = 2 \times 1/3$). (*Progressions for the Common Core State Standards in Mathematics 3–5, Number and Operations—Fractions,* pp. 9–11) Students multiplied a whole number up to four digits by a one-digit whole number and multiplied two-digit numbers, using strategies based on place value and properties of operations. They understood a fraction a/b as a multiple of $1/b$ using a visual fraction model. Students used their understanding to multiply a fraction by a whole number. They solved word problems using multiplication of a fraction by a whole number.

Current Learning

In grade 5, students add and subtract fractions with unlike denominators using equivalent fractions to produce like denominators. They solve real-world problems using visual fraction models, equations, benchmark fractions, and number sense.

Students apply and extend their understandings by interpreting a fraction as division of the numerator by the denominator. They multiply a fraction or whole number by a fraction. Students use visual fraction models to show (e.g., $2/3 \times 4 = 8/3$) and create a story context to interpret products as part of a partition.

Fifth graders find the area of a rectangle with fractional side lengths. They interpret multiplication as scaling (resizing).

Students solve real-world problems with multiplication of fractions and mixed numbers. They divide fractions by whole numbers and whole numbers by fractions. Students solve real-world problems using division of unit fractions by whole numbers and division of whole numbers by unit fractions.

Future Learning

In grade 6, students will solve word problems involving division of fraction by fractions, using visual fraction models and equations. Sixth graders will understand the concept of a ratio, associate a unit rate a/b with $a:b$ with $b \neq 0$, and use rate language. They will use rate and rate reasoning to solve real-world problems.

Additional Findings

In grade 5, students need not express the formula in general algebraic form, but rather reason out many examples using fraction strips and number line designs for more complicated examples an area model is useful.

Students work with multiplying by unit fractions and interpreting fractions in terms of division, enables students to see that multiplying a quantity by a number smaller than one produces a smaller quantity.

Progressions for the Common Core State Standards in Mathematics 3–5, Number and Operations—Fractions (pp. 9–11)