

Grade 6 Mathematics, Quarter 2, Unit 2.1

Measurement with Ratios

Overview

Number of instructional days: 15 (1 day = 45 minutes)

Content to be learned

- Use ratio reasoning to solve real-world and mathematical problems.
- Use rate reasoning to solve real-world and mathematical problems.
- Use ratio and rate reasoning to convert measurement units.
- Use appropriate labels when naming units of measure.

Essential questions

- How are ratios used to convert metric measurements?
- How are ratios used to convert customary measurements?

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Consider the units involved and attend to the meaning of quantities, not just how to compute them.
- Make sense of quantities and their relationships.

Look for and express regularity in repeated reasoning.

- Look for mathematically sound shortcuts.
- Continually evaluate the reasonableness of results.

- How does using ratios to convert measurements in the metric system differ from using ratios to convert measurements in the customary system?
- How do you select appropriate units of measure?

Written Curriculum

Common Core State Standards for Mathematical Content

Ratios and Proportional Relationships

6.RP

Understand ratio concepts and use ratio reasoning to solve problems.

- 6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In previous learning, students became familiar with standard units in the customary and metric systems. In grade 5, students converted like measurements within a given measurement system. They used these conversions to solve multistep real-world problems.

Current Learning

In grade 6, students are introduced to ratios for the first time. They then use ratio reasoning to convert measurement units and manipulate and transform units appropriately when multiplying and dividing quantities. They use units in both the metric and customary systems.

Future Learning

In grade 7, students will compute and use unit rates that are associated with ratios of fractions, including ratios of length, area, and other quantities measured in like or different units. They will also recognize and represent proportional relationships between quantities.

Additional Findings

According to *A Research Companion's Guide to Principles and Standards for School Mathematics*,

Conventions about units facilitate communication, though arbitrary, standard units (in the customary system) have interesting histories. (Nickerson, 1999)

Measurements with different-sized units imply that different quantities can represent the same measure. These quantities will be inversely proportional to the size of the units. Consequently, a foot-long strip has a measure of 12 inches, or 24 half-inches.

According to *Principles and Standards for School Mathematics*, the study of measurement systems can help students understand aspects of the base-10 system, such as place value. In making conversions, students apply their knowledge of proportions.

Grade 6 Mathematics, Quarter 2, Unit 2.2

Statistics

Overview

Number of instructional days: 15 (1 day = 45 minutes)

Content to be learned

- Understand how asking different types of questions produces statistical variability.
- Display data on a number line, dot plot, histogram, and box plots.
- Understand measures of center (mean, median).
- Develop strategies for summarizing patterns and trends in distributions of data.
- Summarize a data set by reporting observations, describing how it was measured and its unit of measurement.
- Identify outliers based on patterns of center and deviation.
- Identify the most appropriate measure of central tendency based on the data and the context in which the data was gathered.
- Organize, display, and interpret data.

Essential questions

- How can you distinguish between a statistical and nonstatistical question?
- What are the measures of central tendency? How are they used?

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Draw diagrams of important features and relationships.
- Graph data and search for regularity or trends.
- Explain correspondence between tables and graphs.

Model with mathematics.

- Identify important quantities in a practical situation and map their relationships.
- Use tools such as diagrams, tables, and graphs.

Written Curriculum

Common Core State Standards for Mathematical Content

Statistics and Probability

6.SP

Develop understanding of statistical variability.

- 6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.*
- 6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
- 6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

- 6.SP.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
- 6.SP.5 Summarize numerical data sets in relation to their context, such as by:
- a. Reporting the number of observations.
 - b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
 - c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
 - d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and

relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In grade 3, students created picture graphs and scaled bar graphs to represent data. They solved one- and two-step “how many more” problems. In grade 4, students made line plots to display data sets of measurements given in fractions of units of data. They solved addition and subtraction problems related to the data. In grade 5, students created line plots in fractions of a unit. They used operations on fractions to solve problems.

Students have developed an understanding of decimals, fractions, and whole numbers and the properties of operations to add and subtract.

Current Learning

In grade 6, students are introduced to statistics. They investigate appropriate statistical questions: “How old are the students in my school?” rather than “How old am I?” Students recognize that different measures of central tendency yield different values. They also recognize that a measure of variability (range or standard deviation) can be used to summarize data.

Future Learning

In grade 7, students will use a random sampling to draw inferences about a population. They will compare two data distributions and address questions about differences between populations.

Additional Findings

According to *Principles and Standards for School Mathematics*,

Students need to understand that the mean “evens out” or “balances” a set of data and that the median identifies the “middle” of a data set. The teacher has an important role in providing experiences that help students construct a solid understanding of the mean and its relation to other measures of center.

Misconceptions about probability have been held not only by many students but also by many adults. To correct misconceptions, it is useful for students to make predictions and then compare the predictions with actual outcomes. Computer simulations may help students avoid or overcome erroneous probabilistic thinking.

According to *A Research Companion to Principles and Standards for School Mathematics*, teachers must provide students with alternative ways of approaching data and chance—alternatives to a subjective “bottom line hunches and beliefs” approach.

The Number System in the Coordinate Plane

Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned

- Use positive and negative numbers to represent quantities in a real-world context.
- Recognize numbers with opposite signs as locations of that number on opposite sides of zero on the number line.
- Understand that integers are a set of all positive whole numbers and their opposites and that zero is its own opposite.
- Find and position integers and other rational numbers on a horizontal or vertical number line.
- Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane.
- Find and position pairs of integers and other rational numbers on a coordinate plane.
- Solve real-world and mathematical problems by graphing points in all quadrants of the coordinate plane.
- Draw polygons in the coordinate plane and find the lengths of side when coordinates have the same x - or y -values.

Essential questions

- How do you model a zero pair on a number line?
- How can you plot ordered pairs of rational numbers on a coordinate plane?
- Given a coordinate plane, what is the process for plotting ordered pairs?

Mathematical practices to be integrated

- Look for and make use of structure.
- Rely on using concrete objects or pictures to help conceptualize and solve a problem.
 - Graph data.
- Model with mathematics.
- Identify important quantities in a practical situation.
 - Map their relationships using such tools as diagrams and graphs.

Written Curriculum

Common Core State Standards for Mathematical Content

The Number System

6.NS

Apply and extend previous understandings of numbers to the system of rational numbers.

- 6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
- 6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
- a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.
 - b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
 - c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
- 6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Geometry

6.G

Solve real-world and mathematical problems involving area, surface area, and volume.

- 6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Common Core Standards for Mathematical Practice

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line

for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In previous grades, students explored number lines and coordinate planes. In grade 5, they extended their previous work to use two perpendicular number lines to define a coordinate system.

Current Learning

In grade 6, students understand that positive and negative integers represent quantities having opposite directions or values (e.g., temperature: below/above zero; credits/debits). They use positive and negative integers to represent quantities in a real-world situation.

Students plot points on a number line and ordered pairs on a coordinate plane. They solve real-world problems by graphing points in all four quadrants and find distances between points that have the same first or second coordinate. Students also draw polygons, given coordinates for the vertices.

Future Learning

In grade 7, students will add, subtract, multiply, and divide with the system of rational numbers. They will represent addition and subtraction on a horizontal or vertical number line diagram.

Additional Findings

According to *Principles and Standards of Mathematical Practice*, positive and negative integers should be seen as useful for noting relative changes or values. Students can also appreciate the utility of negative integers when they work with equations whose solution requires them.

