Grade 8 Mathematics, Quarter 1, Unit 1.1

# Understanding and Using Rational and Irrational Numbers

## Overview

**Number of instructional days:** 10 (1 day = 45 minutes)

### Content to be learned

- Convert fractions (and mixed numbers) to decimals.
- Change repeating decimals to fractions.
- Discover the meaning of an irrational number.
- Find rational approximations of irrational numbers and use those approximations to compare the size of irrational numbers.
- Approximate the location of irrational numbers on the number line.
- Find square roots of small perfect squares (1, 4, 9, 16, … , 144).
- Investigate the area of squares ($x^2 = p$).
- Find cubed roots of small perfect cubes (1, 8, 27, 64, 125).
- Investigate the volume of cubes ($x^3 = p$).

### Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Create a representation of the area of a square and volume of a cube to find square and cubed roots, respectively.

Use appropriate tools strategically.

- Use knowledge of perfect squares to find rational approximations of irrational numbers.

Attend to precision.

- Calculate accurately and efficiently.

### Essential questions

- What are different ways to represent rational numbers?
- How do you distinguish between rational and irrational numbers?
- How can you make a rational approximation of an irrational number?
- What is the relationship between squares and cubes?

# Written Curriculum

## Common Core State Standards for Mathematical Content

| The Number System | 8.NS |
|---|---|

**Know that there are numbers that are not rational, and approximate them by rational numbers.**

8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., $\pi^2$). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

| Expressions and Equations | 8.EE |
|---|---|

**Work with radicals and integer exponents.**

8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

## Common Core Standards for Mathematical Practice

**2 Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

**5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that

technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## Clarifying the Standards

*Prior Learning*

In grade 6, students gained an understanding of numbers and ordered rational numbers on a number line. They also multiplied and divided fractions and decimals with fluency. Students wrote and evaluated expressions involving whole number exponents. Students used the formulas for the volume of a cube and area of a square ($V = s^3$ and $A = s^2$), and they learned about area and volume of geometric figures. In grade 7, students applied and extended multiplication and division of fractions to multiply and divide rational numbers. Students converted rational numbers to decimals using long division, and they learned that the decimal expansion of a rational number terminates in zeros or eventually repeats.

*Current Learning*

Students convert fractions to decimals (reinforced concept). They discover the meaning of irrational numbers and the radical symbol for the first time. Students learn that every number has a decimal expansion. They convert repeating decimals into fractions. Students use the square root and cube root symbols to represent solutions to equations in the form $x^2 = p$ and $x^3 = p$ where $p$ is a positive, rational number. They evaluate square roots of small perfect squares and cube roots of small perfect cubes.

*Future Learning*

In Algebra 1, students will extend their knowledge of integer exponents to include the properties of rational exponents. They will extend their knowledge of rational and irrational numbers by using the properties to add, subtract, multiply, and divide them.

## Additional Findings

Students often have misconceptions resulting from not knowing their benchmark fractions and decimals and lacking fluency in expressing numbers in different forms (e.g., 4 = 8/2 or 1.2 = 12/10 = 1-1/5). They also have misconceptions regarding how to express repeating decimals as rational numbers, knowing that the square root of nonperfect squares are always irrational, and distinguishing area and volume and knowing their differences.

Grade 8 Mathematics, Quarter 1, Unit 1.2

# Properties of Translation, Rotation, and Reflection of Two-Dimensional Figures

## Overview

**Number of instructional days:** 15 (1 day = 45 minutes)

### Content to be learned

- Identify corresponding line segments and angles in two different figures.
  - Recognize that corresponding angles and sides are congruent for rotations, reflections, and translations, but not dilations.
- Describe how the size of a figure is affected by a dilation.
  - Understand that in a dilation corresponding sides are proportional and corresponding angles are congruent.
  - Understand that the image formula for a dilation is $d(x, y) = (kx, ky)$, where $k$ is a real number, called the *magnitude* or *scale factor*, and where the center of the dilation is the origin.
- Recognize that the translation image formula has the form $T(x, y) = (x + a, y + b)$, where $a$ is the number of units moved horizontally and $b$ is the number of units moved vertically.
- Understand that the image formula for a 90° rotation is $R(x, y) = (-y, x)$.
- Understand that the reflection over the $x$-axis is $r(x, y) = (x, -y)$, where $r$ represents the reflection, and that the reflection over the $y$-axis is $r(x, y) = (-x, y)$, where $r$ represents the reflection.

### Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Use concrete objects and/or pictures to help conceptualize transformations.

Look for and make use of structure.

- Look closely to discern a pattern in the outcome of the coordinates of points after completing a transformation.
- Look for, develop, generalize, and describe a pattern orally, symbolically, graphically, and in written form.

### Essential questions

- What connections exist between transformations and congruence/similarity?
- How would you describe the differences among translations, rotations, reflections, and dilations?
- How can you determine what transformation was performed by analyzing the original set of points and the points of the transformed figure?

## Written Curriculum

### Common Core State Standards for Mathematical Content

| **Geometry** | **8.G** |
|---|---|

**Understand congruence and similarity using physical models, transparencies, or geometry software.**

8.G.1 Verify experimentally the properties of rotations, reflections, and translations:

a. Lines are taken to lines, and line segments to line segments of the same length.

b. Angles are taken to angles of the same measure.

c. Parallel lines are taken to parallel lines.

8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

### Common Core Standards for Mathematical Practice

**1 Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

**3 Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

## Clarifying the Standards

*Prior Learning*

Since as early as grade 4, students studied the properties of parallel lines, line segments, and angles. In subsequent grades, they learned how to graph points on the coordinate plane. Students mastered drawing, constructing, and describing geometric figures and explained the relationships between them.

*Current Learning*

Students discover that any two-dimensional figure can be mapped onto another through a series of transformations. They express a transformation as a mapping from one set of coordinates to another. Students know that a two-dimensional figure is congruent to another if the second figure can be obtained from a sequence of rotations, reflections, and translations. They learn the effect of dilations, rotations, reflections, and translations on their coordinates.

*Future Learning*

In Geometry, students will develop precise definitions of points, lines, planes, segments, and angles. They will develop an understanding of two-dimensional congruence through various types of transformations. Students will learn to express a transformation mapping as a function rule and a sequence of transformations. In Geometry, they will also determine definitions of geometric terms to prove geometric theorems.

## Additional Findings

The use of manipulatives is critical for the understanding of this unit. Students have a difficult time between clockwise and counterclockwise rotation. (It is much easier to rotate about a point. Make sure that students have a firm grasp on graphing ordered pairs [may be a minilesson].)

The following is a useful website: www.bookrags.com/research/transformations-mmat-04.

Technology assists students in understanding the visual connection of translations (e.g., software applications). Translations are used in the real world (e.g., cameras, graphic arts, computer games, construction).

Grade 8 Mathematics, Quarter 1, Unit 1.3

# Proving Congruence and Similarity Using the Properties of Dilation, Translation, Rotation, and Reflection

## Overview

**Number of instructional days:** 15 (1 day = 45 minutes)

### Content to be learned

- Use the image formula (see Unit 1.2 content to be learned) for a dilation to determine similarity.
- Use the image formulas (see Unit 1.2 content to be learned) for translations, reflections, and/or rotations to determine if two figures are congruent.
- Write a sequence of steps to show congruence/ similarity from figure A to A'.

### Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Identify and execute appropriate transformation strategies as taught in the previous unit.

Construct viable arguments and critique the reasoning of others.

- Use prior learning of transformation formulas taught in Unit 1.1 in constructing arguments.
- Communicate conclusions to others by showing the series of transformations, and respond to the arguments of others.

### Essential questions

- What connections exist between transformations and congruence?
- How would you describe the differences among translations, rotations, reflections, and dilations?
- How can you determine what transformation was performed by analyzing the original set of points and the points of the transformed figure?

# Written Curriculum

## Common Core State Standards for Mathematical Content

| Geometry | 8.G |
|---|---|

**Understand congruence and similarity using physical models, transparencies, or geometry software.**

8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

## Common Core Standards for Mathematical Practice

**1 Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

**3 Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an

argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

## Clarifying the Standards

*Prior Learning*

Since as early as grade 4, students studied the properties of parallel lines, line segments, and angles. In subsequent grades, they learned how to graph points on the coordinate plane. Students mastered drawing, constructing, and describing geometric figures and explained the relationships between them. In the previous unit, students discovered that any two-dimensional figure can be mapped onto another through a series of transformations.

*Current Learning*

Students understand that a two-dimensional figure is congruent to another (using physical models, transparencies, or geometry software) if the second can be obtained from the first by a sequence of reflections, rotations, or translations. Students also understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of reflections, rotations, translations, and dilations.

*Future Learning*

In Geometry, students will develop precise definitions of points, lines, planes, segments, and angles. They will develop an understanding of two-dimensional congruence through various types of transformations. Students will learn to express a transformation mapping as a function rule and a sequence of transformations. In Geometry, students will also use definitions of geometric terms to prove geometric theorems.

## Additional Findings

The use of manipulatives and geometry software is critical for the understanding of this unit. Students need to understand rules for similarity (corresponding angles are congruent, corresponding sides are proportional).