Grade 8 Mathematics, Quarter 2, Unit 2.1

Angle Measure Facts and Interior and Exterior Angle Relationships to Parallel Lines Cut by a Transversal

Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned

- Understand and verify informally that the sum of the measures of interior angles of any triangle is 180 degrees.
- Understand and verify informally that the measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles of the triangle.
- Understand and verify informally the properties of angles formed when two parallel lines are cut by a transversal.
- Understand and verify informally the angle-angle criterion for similarity of triangles.

Mathematical practices to be integrated

Model with mathematics.

- Read, understand, and draw conclusions about angle relationships when a pair of parallel lines is cut by a transversal.
- Interpret results in the context of a real-world problem and make revisions if necessary.

Use appropriate tools strategically.

- Use geometry software to solve problems, check solutions, visualize results, and compare predictions.
- Make sound decisions about when to use physical models, transparencies, and/or geometry software.

Look for and make use of structure.

- Look for a pattern or structure to discover the properties of parallel lines cut by a transversal.
- Look for relationships among the angles of a triangle.
- See how the exterior angles of a triangle are related to the interior angles.

Essential questions

- What are the relationships between any of the angles formed by two parallel lines cut by a transversal?
- How are the exterior angles and interior angles of any triangle related?
- What connections exist between congruence and similarity in triangles?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry	8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so*.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In grade 7, students used facts about supplementary, complementary, vertical, and adjacent angles to solve real-world problems. In Unit 1.3, students learned the concepts of congruence and similarity with two-dimensional figures.

Current Learning

Students informally establish facts about angle sum and exterior measures of triangles. They also learn about the angles created when parallel lines are cut by a transversal. Students discover similarity of triangles by the angle-angle criteria.

Future Learning

In Geometry, students will prove theorems about lines and angles. They will prove angle congruence by using their prior knowledge of parallel lines cut by a transversal. Students will prove theorems about triangles.

Additional Findings

Students may not understand that a straight line is 180 degrees. If you put the three angles of a triangle together, they form a straight line.

English language learners/special education students may have a tough time with the vocabulary of the angle relationships.

Grade 8 Mathematics, Quarter 2, Unit 2.2

Understanding and Applying the Pythagorean Theorem

Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned

- Prove the Pythagorean Theorem and its converse.
- Determine the side lengths of right triangles by applying the Pythagorean Theorem.
- Apply the Pythagorean Theorem in real-world problems in both two and three dimensions.
- Find the distance between two points on the coordinate plane using the Pythagorean Theorem.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Create a representation to illustrate why the sum of the squares of the legs of a right triangle are equivalent to the square of the hypotenuse.
- Understand the quantities and relationships of the legs and hypotenuse of a right triangle.

Model with mathematics.

- Create a two-dimensional cross section from a three-dimensional model to analyze and draw conclusions regarding right triangles.

Attend to precision.

- Solve algebraic equations accurately and efficiently by using the Pythagorean Theorem with proper units.

Essential questions

- How is the Pythagorean Theorem useful in finding the side lengths of a right triangle?
- How could you use the Pythagorean Theorem to determine if a structure is a right triangle?
- How can you apply the Pythagorean Theorem to find the distance between points on a coordinate plane?
- How can the Pythagorean Theorem be used to solve real-world problems involving two and three dimensions?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry 8.G

Understand and apply the Pythagorean Theorem.

8.G.6 Explain a proof of the Pythagorean Theorem and its converse.

8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about

specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

Students recognized right triangles as early as grade 4. In grade 7, students constructed triangles with three measures of angles or sides, noticing when the conditions determine a unique triangle. They also worked with three-dimensional figures. Since grade 5, students also used the coordinate plane and graphed ordered pairs. Since grade 6, students have solved algebraic equations. In Unit 1.1, students learned how to calculate square roots.

Current Learning

Students use their prior knowledge of right triangles to discover and prove the Pythagorean Theorem and its converse. They determine the side lengths of right triangles by applying the Pythagorean Theorem, while using it to solve real-world problems in both two and three dimensions. Students apply their findings to find the distance between two points in a coordinate plane and also in a three-dimensional model.

Future Learning

In Geometry, students will prove theorems about triangles. The Pythagorean Theorem will be proven using triangle similarity. Students will also use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Additional Findings

On the coordinate plane, if legs of a right triangle are not parallel to the axes, students may have to circumscribe a rectangle about the triangle.

To further student understanding, consider introducing students to the distance formula.

Real-world examples of Pythagorean Theorem can be found at www.keypress.com/documents/ALookInside/DiscoveringGeometry/DG_TE_Ch09.pdf.

Worksheet Generator for practice on Pythagorean Theorem can be found at http://www.math-aids.com/Pythagorean_Theorem.

The Spider and the Fly example (three-dimensional example of Pythagorean Theorem) can be found at http://nrich.maths.org/2365/2365.

Grade 8 Mathematics, Quarter 2, Unit 2.3

Solving Real-World Problems Involving Volume of Cones, Cylinders, and Spheres

Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned

- Demonstrate knowledge of the formulas for the volumes of cylinders, cones, and spheres.
- Apply these formulas to solve real-world mathematical problems.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Create a representation of a problem using a cone, cylinder, or sphere.
- Consider the units and attend to the meaning of the units when finding the volume of each figure.

Attend to precision.

- Use accurate math language and units of measure when communicating reasoning or solutions to a volume problem.
- Calculate accurately and efficiently when using appropriate volume formulas.

Essential questions

- How is a formula helpful in finding the volume of cylinders, cones, and spheres?
- What connections exist between the formulas for the volumes of cylinders, cones, and spheres?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry 8.G

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 5, students learned the concepts of volume in relation to multiplication and addition. In grades 6 and 7, students solved real-world and mathematical problems involving area, surface area, and volume. They also used expressions and equations to solve real-world and mathematical problems using numerical and algebraic expressions.

Current Learning

Students learn the formulas for volume of cylinders, cones, and spheres and use them to solve real-world and mathematical problems.

Future Learning

In Geometry, students will explain volume formulas and use them to solve problems. They will visualize relationships between two- and three-dimensional objects. Students will apply geometric concepts in modeling concepts. They will use geometric shapes, their measures, and their properties to describe objects.

Additional Findings

Students have a difficult time understanding the concept of a cubic unit. They do not often see the relationship between area and volume.

Video on the relationships between volumes of cylinders, cones, and spheres can be found at www.youtube.com/watch?v=NAcTBJ1boD4.

Grade 8 Mathematics, Quarter 2, Unit 2.4

Using Radicals, Integer Exponents, Expanded and Scientific Notation to Represent Values

Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned

- Know and apply the properties of integer exponents.
- Use square and cube root symbols to represent solutions to equations.
- Evaluate square roots of small perfect squares and cube roots of small perfect cubes.
- Know that the $\sqrt{2}$ is irrational.
- Change numbers that are in standard form to scientific notation.
- Change numbers that are in scientific notation to standard form.
- Estimate very large or very small quantities when written as a number times a power of 10.
- Use real-world examples of very large or very small quantities to explore real-life applications of scientific notation.

Mathematical practices to be integrated

Look for and make use of structure.

- Recognize a pattern as an exponent increases or decreases.
- Look for a relationship between positive and negative exponents.

Look for and express regularity in repeated reasoning.

- Look for general methods and shortcuts once a pattern is recognized.
- Notice if calculations are repeated.

Essential questions

- How can rational numbers be expressed with exponents?
- What is the relationship between positive and negative exponents?
- How can equivalent expressions be written in scientific notation and standard form?
- How are very small and very large numbers represented in scientific notation?
- How does a pattern in the exponent relate to a geometric sequence?
- How is scientific notation applied to real-world situations?

Written Curriculum

Common Core State Standards for Mathematical Content

Expressions and Equations 8.EE

Work with radicals and integer exponents.

8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.*

8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Common Core Standards for Mathematical Practice

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In grades 6 and 7, students solved mathematical problems using exponents. In the early part of grade 8, students found square roots of small perfect squares and cube roots of small perfect cubes.

Current Learning

Students use their knowledge of integer exponents to evaluate equations using square and cube roots. They extend their knowledge by rewriting numbers that are in standard form into scientific notation. Students use scientific notation to make accurate estimates of very large or very small numbers. They learn that any change in the exponent renders a significant change in value when comparing two quantities in scientific notation.

Future Learning

In Algebra 1, students will extend their knowledge by graphing square and cube root functions. They will solve and graph exponential and logarithmic functions. Students will use the properties of exponents to interpret expressions for exponential functions, classifying them as representing exponential growth and decay.

Additional Findings

Students have misconceptions with negative exponents; most believe that a negative exponent has a negative value.

Students have a difficult time understanding the value of very small and very large numbers involving scientific notation and exponents.