Algebra 1, Quarter 1, Unit 1.1
Using Units, Interpreting Expressions, and Creating Equations and Inequalities to Solve Problems

Overview

Number of instructional days: 13 (1 day = 45 minutes)

Content to be learned
- Solve multistep equations.
- Use units of measure correctly where appropriate.
- Isolate specific variables in literal equations.
- Given specific data, identify parts of expressions and construct expressions.
- Create, solve, and graph one- and two-variable equations and inequalities.

Mathematical practices to be integrated
Reason abstractly and quantitatively.
- Make sense of quantities and their relationships in problem situations.
- Use varied representations and approaches when solving problems.
- Change perspectives, generate alternatives, and consider different options.

Model with mathematics.
- Apply math skills to solve problems in everyday life.
- Identify important quantities in a practical situation and map their relationships using tools such as diagrams, two-way tables, graphs, flowcharts, and formulas.
- Analyze mathematical relationships to draw conclusions.

Look for and make use of structure.
- Look for, develop, generalize, and describe a pattern orally, symbolically, graphically, and in written form.
- Step back for an overview and shift perspective.
- See complicated things as being composed of several objects.

Essential questions
- Where are linear relationships used in the real world (algebraically and graphically)?
- How do you determine and interpret appropriate scales when constructing graphs of data?
- How do you determine the appropriate precision for a given a set of data?
Written Curriculum

Common Core State Standards for Mathematical Content

<table>
<thead>
<tr>
<th>Number and Quantities*</th>
<th>N-Q</th>
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<tbody>
<tr>
<td>Reason quantitatively and use units to solve problems.</td>
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<tr>
<td>[Foundation for work with expressions, equations, and functions]</td>
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<tr>
<td>N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.*</td>
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<tr>
<td>N-Q.2 Define appropriate quantities for the purpose of descriptive modeling.*</td>
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<tr>
<td>N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*</td>
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<tr>
<th>Seeing Structure in Expressions</th>
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<tbody>
<tr>
<td>Interpret the structure of expressions</td>
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<tr>
<td>[Linear, exponential, quadratic]</td>
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<tr>
<td>A-SSE.1 Interpret expressions that represent a quantity in terms of its context.*</td>
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<tr>
<td>a. Interpret parts of an expression, such as terms, factors, and coefficients.</td>
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Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

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4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

Clarifying the Standards

Prior Learning

In previous grades, students learned to graph data on the coordinate plane (in some cases choosing and interpreting the scale and specific intervals). In grade 7, students mastered identifying the parts of an expression and how to solve and graph inequalities on a number line. In addition, eighth graders learned about basic units of measure and how to solve equations.

Current Learning

Students learn how to use the context of a math problem to guide the solution of multistep problems. They also construct linear equations and inequalities to solve and interpret solutions with both one and two variables.

Future Learning

Students will use this problem-solving technique of using the context of a problem to help understand quadratic and exponential functions in later units of Algebra 1. Knowledge of initial linear equations will help forge a relationship between standard form, point-slope form, and slope-intercept form.
Additional Findings

*Science for All Americans* states, “A central line of investigation in theoretical math is identifying each field of study a small set of basic ideas and rules which all other interesting ideas and rules in that field can be logically deduced. Mathematicians, like other scientists, are particularly pleased when previously unrelated parts of math are found to be derivable from one another, or from some more general theory.” (p. 16)

Teachers should stress to learners the idea that \((x, y)\) is not just a numerical answer, but it has context as an actual solution to an actual problem with units of measure. Additionally, students may find difficulty with manipulating equations to solve for a specific variable.

Common student misconceptions are that all linear equations have a \(y\)-intercept and that all equations can be written in slope-intercept form (i.e., \(x = 4\)). Students also many times confuse the \(y\)-value as the \(y\)-intercept when the equation is written in slope-intercept form.

When teaching graphing equations in two variables, the focus is on recognizing patterns and using a table of values.
Algebra 1, Quarter 1, Unit 1.2
Solving and Justifying Solutions to Equations and Inequalities

Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned
- Solve multistep equations/inequalities and explain each step.
- Solve problems in two variables and graph the solution set.
- Explain that the graph of an equation in two variables is the set of all its solutions.
- Manipulate formulas to solve for a specific variable.

Mathematical practices to be integrated
- Attend to precision.
- Communicate reasoning or solutions and use accurate math language.
- Calculate accurately and efficiently.

Essential questions
- How can you represent the same equation in multiple ways?
- In what ways can you justify the process of solving an equation/inequality?
- How is an equation related to its graph?
Written Curriculum

Common Core State Standards for Mathematical Content

<table>
<thead>
<tr>
<th>Reasoning with Equations and Inequalities</th>
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<tr>
<td>Understand solving equations as a process of reasoning and explain the reasoning [<em>Master linear; learn as general principle</em>]</td>
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<tr>
<td>A-REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.</td>
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| Solve equations and inequalities in one variable [*Linear inequalities; literal that are linear in the variables being solved for; quadratics with real solutions*] |
| A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. |

| Represent and solve equations and inequalities graphically [*Linear and exponential; learn as general principle*] |
| A-REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). |

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<td>A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law $V = IR$ to highlight resistance $R$.*</td>
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Common Core Standards for Mathematical Practice

6 **Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

8 **Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention

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to the calculation of slope as they repeatedly check whether points are on the line through \((1, 2)\) with slope 3, middle school students might abstract the equation \((y - 2)/(x - 1) = 3\). Noticing the regularity in the way terms cancel when expanding \((x - 1)(x + 1)\), \((x - 1)(x^2 + x + 1)\), and \((x - 1)(x^3 + x^2 + x + 1)\) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

**Clarifying the Standards**

**Prior Learning**

In prior grades, students worked with and mastered solving multistep linear equations and inequalities. They also learned that solutions to linear equations lie on the graphed line and that there are an infinite number of solutions to this representation (continuous versus discrete).

Students developed an understanding of rewriting an expression in different forms, how it can shed light on the problem, and how the quantities in it are related.

**Current Learning**

Students broaden their knowledge through reinforcement of solving equations/inequalities with real-world examples, and they interpret the meaning of the solution(s). With the rearrangement of formulas, students highlight a quantity of interest and understand its relationship to the other terms in the formula.

**Future Learning**

Students will use the above skills in future Algebra 1 units, across the mathematical content area, and in science courses.

**Additional Findings**

Students often have a difficult time applying the principles of solving an equation to formulas with variables only. Fully grasping that the idea of infinite solutions to an equation can be challenging for students. Teachers may find it advantageous to help students make the connections between a graph and an equation by using a table of values (which will be covered in Unit 1.4).

It may be beneficial to have students solve equations using a two-column algebraic proof format, using properties of real numbers. This practice provides students with exposure to “proofs,” which will help them in Geometry.
Algebra 1, Quarter 1, Unit 1.3
Creating and Solving Linear Equations and Inequalities

Overview

Number of instructional days: 8 (1 day = 45 minutes)

Content to be learned

• Create equations in slope-intercept, point-slope, and standard forms from real-world applications, graphs, and tables.
• Identify the x- and y-intercepts.
• Solve and graph equations/inequalities.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.
• Analyze given information to develop possible strategies for solving the problem.

Attend to precision.
• Calculate accurately and efficiently.
• Formulate explanations clearly when communicating reasoning to others.

Essential questions

• How can you write a linear equation to represent a real-world situation?

• In what ways can you use different forms of linear equations to identify specific attributes of the relationship (i.e., slope, intercepts)?
• How would you graph a linear relationship?
Written Curriculum

Common Core State Standards for Mathematical Content

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<td>A-REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.</td>
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Common Core Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 7, students constructed and graphed simple equations/inequalities to solve problems and interpreted the solution set in the context of the problem. In grade 8, students constructed functions to model linear relationships between two quantities. They also determined the rate of change and initial value of functions from a description of a relationship or from two \((x, y)\) values. Previously in this course, students created and graphed expressions using a table of values.

Current Learning

Students develop an understanding of linear relationships as they create and graph linear equations/inequalities in various forms: slope-intercept, point-slope, and standard. In addition, students identify the solution set.

Future Learning

Later in this course, students will use their knowledge of linear relationships to determine domain and range and when using function notation in Unit 1.4. The same skills will be used when students solve a system of equations.

Additional Findings

Graphing lines in two variables in this unit differs from Unit 1.1 in that it focuses on utilizing slope-intercept form rather than using a table of values.

Teachers should be cognizant that despite having worked with slope as a rate of change in grade 7, a review of the slope formula may be necessary.

Students need to understand that all forms of linear equations are related.

Teachers should stress in which situations one form of a linear equation would be advantageous to use over another. (i.e., When graphing a line, slope-intercept would be best; however, when trying to find the intercepts, perhaps standard form would be best. When given slope and a point, point-slope form may be easiest to use.)
Algebra 1, Quarter 1, Unit 1.4
Understanding Functions and Function Notation and Interpreting the Intersection of the Graphs of Two Functions

Overview

Number of instructional days: 4  (1 day = 45 minutes)

Content to be learned
- Find the slope of a linear function.
- Express the domain and range of a linear function in set builder notation.
- Evaluate functions.
- Interpret statements that use function notation in terms of a context.
- Recognize that sequences are functions.
- Graph equations and inequalities in function form (i.e., using a table of values, technology, or finding successive approximations).
- Interpret the meaning of the intersection of two lines on a graph.

Mathematical practices to be integrated
Model with mathematics.
- Read, understand, and draw conclusions from graphs, charts, tables, etc.
- Interpret results in the context of the problem situation and make revisions, if necessary.

Attend to precision.
- Calculate accurately and efficiently.
- When communicating their reasoning or solutions, use accurate math language, units of measure, labeling axes, etc.

Essential questions
- How do you write linear relationships in functional notation?
- How is function notation useful in identifying a solution?
- How do you define the domain and range of a function?
- What does the intersection of two lines on a graph represent?
Written Curriculum

Common Core State Standards for Mathematical Content

Interpreting Functions

**F-IF**

**Understand the concept of a function and use function notation** [Learn as general principle; focus on linear and exponential and on arithmetic and geometric sequences]

**F-IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \).

**F-IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

**F-IF.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by \( f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) \) for \( n \geq 1 \).

Reasoning with Equations and Inequalities

**A-REI**

**Represent and solve equations and inequalities graphically** [Linear and exponential; learn as general principle]

**A-REI.11** Explain why the \( x \)-coordinates of the points where the graphs of the equations \( y = f(x) \) and \( y = g(x) \) intersect are the solutions of the equation \( f(x) = g(x) \); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where \( f(x) \) and/or \( g(x) \) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.

Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
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Clarifying the Standards

Prior Learning

In grade 8, students developed their knowledge of how to define, evaluate, and compare functions. They used functions to model relationships between quantities.

Current Learning

Students reinforce their knowledge of rate of change and initial value of functions from a description of a relationship of two points \((x_1, y_1)\) and \((x_2, y_2)\). They begin to develop knowledge of functional notation and develop an understanding that the domain is the set of input values and that the range is the set of output values of a function.

Future Learning

Students will reinforce their knowledge in future units of this course, including Unit 2.1 and with nonlinear functions in Unit 4.1. Domain and range will be utilized for much of the rest of this Algebra 1 course and in Algebra 2.

Additional Findings

Teachers should reinforce that \(f(x)\) does not mean \(f\) multiplied by \(x\).

Consider differentiating instruction when determining the domain and range of a graphed function (i.e., use of manipulatives to isolate one axis at a time; discuss the end behavior of the graph).

Identify the domain as the independent variable and the range as the dependent variable.