Overview

Number of instructional days: 7  (1 day = 45 minutes)

Content to be learned
• Use function notation.
• Sketch and identify key characteristics of the graphs of linear functions.
• Use the domain of a function in relation to the context.
• Calculate and interpret the rate of change of a linear function over a specified interval.

Mathematical practices to be integrated
Model with mathematics.
• Apply math skills to solve problems in everyday life.
• Identify important quantities using relevant data in a word problem.
• Analyze, draw conclusions, and interpret results in context (collect data, create graphs, explain results).

Making sense of problems and persevere in solving them.
• Understand and explain the meaning of a problem.
• Plan a solution pathway given constraints, relationships, and goals.

Attend to precision.
• When communicating reasoning or solutions, use accurate math language, units of measure, labeling axes, etc.
• Use clear definitions and state the meaning of symbols.
• Calculate accurately and efficiently.

Essential questions
• What information about a graph can you extract given the equation of a function?
• How can a graph or table be used to express the rate of change of a function?
• What does the domain of a function say about the data being represented?
• What conceptual vocabulary helps you describe the graph of a function?
Written Curriculum

Common Core State Standards for Mathematical Content

Interpreting Functions

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions
and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6   **Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

**Clarifying the Standards**

*Prior Learning*

In grade 8, students learned that a function is a rule that assigns each input exactly one output. They learned that the equation \( y = mx + b \) defines a linear function whose graph is a straight line that is a set of ordered pairs. In Unit 1.1, students further reinforced their ability to analyze and construct functions to model a linear relationship between two quantities, interpreting the rate of change and initial value of the function in terms of the situation it models and in terms of its graph or table of values.

*Current Learning*

Students are asked to interpret key features of graphs or tables in terms of the quantities and to sketch a graph based on a verbal description. They are introduced to function notation and relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Students calculate and interpret the average rate of change over a specified interval. In this unit, functions are modeling contextual situations.

*Future Learning*

In the next unit, students will develop the ability to graph, create, compare, and use functions to model situations. They will draw from their knowledge of analyzing these graphs in later units. Also, in Unit 2.4, students will analyze and construct graphs of quadratic and logarithmic functions, so they will compare the key features of the linear functions to those of the new functions.

**Additional Findings**

Teachers may find that additional instruction is needed if students are unable to demonstrate fluidity between multiple representations with linear relationships (graphs, tables, equations).

Teachers should ensure that students can use and understand conceptual vocabulary when describing functions (intercepts, intervals, increasing, decreasing, positive, negative, etc.).
Algebra 1, Quarter 2, Unit 2.2
Graphing, Creating, Comparing, and Using Functions to Model Situations

Overview

Number of instructional days: 8
(1 day = 45 minutes)

Content to be learned
- Interpret the parameters in a linear function in terms of the context.
- Extract relevant information from word problems.
- Build functions from word problems.
- Compare linear functions to identify differences.
- Graph linear and piecewise functions.
- Identify patterns in data tables and word problems; use them to construct a function; and when given a function, construct a table of values.

Mathematical practices to be integrated
Model with mathematics.
- Apply prior knowledge to solve problems arising in everyday life, society, and the workplace.
- Read, understand, and draw conclusions from graphs, charts, tables, etc.
- Interpret results in the context of the problem and make revisions if necessary.

Attend to precision.
- Communicate precisely using mathematical vocabulary.
- When communicating reasoning or solutions, use units of measure, labeling axes, etc.

Essential questions
- How can you determine the best representation (graph, table, equation) to use for situational linear functions?
- What key features are necessary to construct a linear function?
Written Curriculum

Common Core State Standards for Mathematical Content

**Interpreting Functions**  
F-IF

Analyze functions using different representations [*Linear, exponential, quadratic, absolute value, step, piecewise-defined]*

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

**Building Functions**  
F-BF

Build a function that models a relationship between two quantities [*For F.BF.1, 2, linear, exponential, and quadratic]*

F-BF.1 Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*

Build new functions from existing functions [*Linear, exponential, quadratic, and absolute value; for F.BF.4a, linear only]*

F-BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

**Linear, Quadratic, and Exponential Models***  
F-LE

Construct and compare linear, quadratic, and exponential models and solve problems

F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*

Cranston Public Schools, with process support from the Charles A. Dana Center at the University of Texas at Austin
Interpret expressions for functions in terms of the situation they model \([\text{Linear and exponential of form } f(x) = b^x + k]\)

F-LE.5 \hspace{1em} Interpret the parameters in a linear or exponential function in terms of a context.*

**Common Core Standards for Mathematical Practice**

4 \hspace{1em} **Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 \hspace{1em} **Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 \hspace{1em} **Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see \(7 \times 8\) equals the well remembered \(7 \times 5 + 7 \times 3\), in preparation for learning about the distributive property. In the expression \(x^2 + 9x + 14\), older students can see the 14 as \(2 \times 7\) and the 9 as \(2 + 7\). They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see \(5 - 3(x - y)^2\) as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers \(x\) and \(y\).
Clarifying the Standards

Prior Learning

In Units 1.4 and 2.1, students developed knowledge of function notation and the key features of a function. They learned to interpret the key components of a graphed situational function.

Current Learning

Students create functions to model real-world situations and graph functions. They form a problem-solving strategy by decontextualizing information from the problem.

Future Learning

In Units 4.2 and 4.3, students will compare, graph, interpret, and classify functions based on their properties.

Additional Findings

Students may have difficulty extracting relevant information within word problems. Extra problems for reinforcement may be needed.
Algebra 1, Quarter 2, Unit 2.3
Using Multiple Methods to Solve Systems of Equations and Inequalities

Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned

- Solve systems of equations using graphs, substitution, and elimination.
- Solve real-world problems involving systems of equations.
- Contextualize the solution given the parameters of a problem.
- Demonstrate understanding of one solution, infinite solutions, and no solution.
- Demonstrate understanding that the solution set of a system of inequalities is the intersection of the shaded regions.

Mathematical practices to be integrated

- Make sense of problems and persevere in solving them.
- Identify and execute appropriate strategies when solving systems of equations.
- Evaluate progress toward a solution and make revisions if necessary.
- Check for the sensibility of a solution, and substitute values into both equations to verify that it is a common solution.

Model with mathematics.

- Apply the concept of solving systems to solve problems arising in everyday life, society, and the workplace.
- Identify important quantities in a practical situation and represent their relationships using diagrams, two-way tables, or graphs to write the linear equations of the system.

Essential questions

- How can strategies for solving systems of equations be used in real-world situations such as maximizing profit/minimizing cost?
- What is the significance of a linear system's solution?
Written Curriculum

Common Core State Standards for Mathematical Content

### Reasoning with Equations and Inequalities  
**A-REI**

#### Solve systems of equations [Linear-linear and linear-quadratic]

**A-REI.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

**A-REI.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

#### Represent and solve equations and inequalities graphically [Linear and exponential; learn as general principle]

**A-REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

### Creating Equations*

**A-CED**

#### Create equations that describe numbers or relationships [Linear, quadratic, and exponential (integer inputs only); for A.CED.3 linear only]

**A-CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

### Common Core Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.

Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In grade 8, students solved systems of equations algebraically and graphically. In Unit 2.1, students mastered graphing linear equations and inequalities in two variables.

Current Learning

Students solve systems of equations by graphing, substitution, and elimination. They also solve a system of inequalities with the solution defined as the intersection of two shaded regions. Students prove that, given a system of two equations of two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

Future Learning

In Unit 4.1, students will apply the concept of solving a system of linear equations to solving a linear-quadratic system of equations. In Algebra 2, students will solve nonlinear systems of equations and inequalities.

Additional Findings

None at this time.
### Algebra 1, Quarter 2, Unit 2.4

**Understanding Rational Exponents and Using Their Properties to Rewrite Expressions**

<table>
<thead>
<tr>
<th>Overview</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of instructional days:</strong> 10</td>
</tr>
<tr>
<td><strong>Content to be learned</strong></td>
</tr>
<tr>
<td>• Use properties of exponents to solve problems with <em>rational</em> exponents.</td>
</tr>
<tr>
<td>• Perform operations with radical expressions.</td>
</tr>
<tr>
<td>• Isolate specific variables within formulas.</td>
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<tr>
<td><strong>Mathematical practices to be integrated</strong></td>
</tr>
<tr>
<td>Construct viable arguments and critique the reasoning of others.</td>
</tr>
<tr>
<td>• Understand and use prior learning of integer exponents to simplify or solve problems with rational exponents.</td>
</tr>
<tr>
<td>Look for and make use of structure.</td>
</tr>
<tr>
<td>• Deconstruct complicated expressions into several small steps in order to isolate a variable.</td>
</tr>
<tr>
<td>• Apply and discuss properties of exponents.</td>
</tr>
</tbody>
</table>

| **Essential questions** |
| • How can you use the properties of exponents to simplify problems with rational exponents? |
| • What are the benefits of isolating a specific variable within a formula? |
Common Core State Standards for Mathematical Content

The Real Number System

Extend the properties of exponents to rational exponents.

N-RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.

N-RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Creating Equations*

Create equations that describe numbers or relationships [Linear, quadratic, and exponential (integer inputs only); for A.CED.3 linear only]

A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law $V = IR$ to highlight resistance $R$.*

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see
complicated things, such as some algebraic expressions, as single objects or as being composed of several
objects. For example, they can see \( 5 - 3(x - y)^2 \) as 5 minus a positive number times a square and use that
to realize that its value cannot be more than 5 for any real numbers \( x \) and \( y \).

**Clarifying the Standards**

*Prior Learning*

In grade 8, students learned to apply the properties of integer exponents to generate equivalent numerical
expressions. For example, \( 3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27} \).

*Current Learning*

Students learn to extend the properties of integer exponents to rational exponents, allowing for radical
notation to be written in terms of rational exponents. For example, you define \( (5^{\frac{1}{3}})^3 \) as the cube root of 5
because you want \( (5^{\frac{1}{3}})^3 \) to hold, so \( (5^{\frac{1}{3}})^3 \) must equal 5. Students solve problems using the properties of
exponents.

*Future Learning*

In Unit 4, students will apply these exponential properties to quadratics and exponential functions. In
Algebra 2, students will relate rational exponents to inverse relations of polynomial functions.

**Additional Findings**

Review operations with exponents of integers.