Algebra 1, Quarter 4, Unit 4.1

Solving Quadratic Equations Using Multiple Methods and Solving Systems of Linear and Quadratic Equations

### Overview

**Number of instructional days:** 13 (1 day = 45 minutes)

**Content to be learned**
- Solve quadratic equations using multiple methods and understand the meaning of the solution(s).
- Graph quadratic equations and their translations.
- Create quadratic equations and interpret the solutions in terms of a real-world problem.
- Solve systems of equations/inequalities involving linear and quadratic functions.

**Mathematical practices to be integrated**
- Make sense of problems and persevere in solving them.
- Consider similar problems.
- Plan a solution pathway by analyzing given constraints, relationships, and goals.
- Reason abstractly and quantitatively.
- Make sense of quantities and relationships within a problem.
- Attend to the meaning of the quantities—not just the computation.
- What is the meaning of a complex solution?
- How does the solution to a system of linear equations compare/contrast to that of a linear and a quadratic system?

**Essential questions**
- How do the different solutions methods of a quadratic equation compare?
- How do you graphically represent the solution(s) to quadratic equations?
### Written Curriculum

**Common Core State Standards for Mathematical Content**

<table>
<thead>
<tr>
<th>Reasoning with Equations and Inequalities</th>
<th>A-REI</th>
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<tr>
<td><strong>Solve equations and inequalities in one variable</strong> [Linear inequalities; literal that are linear in the variables being solved for; quadratics with real solutions]</td>
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<tr>
<td>A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.</td>
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<tr>
<td>A-REI.4 Solve quadratic equations in one variable.</td>
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<tr>
<td>a. Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.</td>
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<tr>
<td>b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers $a$ and $b$.</td>
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<tr>
<td>A-REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.</td>
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<tr>
<td>A-REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</td>
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<tr>
<td><strong>Solve systems of equations</strong> [Linear-linear and linear-quadratic]</td>
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<tr>
<td>A-REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.</td>
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<tr>
<td><strong>Create equations that describe numbers or relationships</strong> [Linear, quadratic, and exponential (integer inputs only); for A.CED.3 linear only]</td>
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</tr>
<tr>
<td>A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*</td>
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<tr>
<td>A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*</td>
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<tr>
<td>A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law $V = IR$ to highlight resistance $R$.*</td>
<td></td>
</tr>
</tbody>
</table>
Interpret functions that arise in applications in terms of the context [Linear, exponential, and quadratic]

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function.*

F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

Build a function that models a relationship between two quantities [For F.BF.1, 2, linear, exponential, and quadratic]

F-BF.1 Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*

Build new functions from existing functions [Linear, exponential, quadratic, and absolute value; for F.BF.4a, linear only]

F-BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Seeing Structure in Expressions A-SSE

Interpret the structure of expressions [Linear, exponential, quadratic]

A-SSE.1 Interpret expressions that represent a quantity in terms of its context.*

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of $P$ and a factor not depending on $P$.

A-SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
Common Core Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Clarifying the Standards

Prior Learning

In unit 3.4, students factored quadratic expressions. Additionally, students learned to solve systems of equations and inequalities in unit 2.3. In unit 1.4, students used function notation to identify specific intervals for domain and range.

Current Learning

Students learn how to solve quadratic equations using factoring, graphing, completing the square, and the quadratic formula. Students solve systems of equations and inequalities (linear and quadratic). Students develop an understanding of the similarities and differences of the graph of a quadratic function versus a linear function (i.e., domain, range, intercepts, intervals, end behavior, translations).

Although there are numerous standards in this unit of study, teachers should focus on applying students’ prior knowledge of linear relationships to quadratic relationships.

Some students may find deriving the quadratic formula helpful in making a connection to the solutions of the quadratic relationship. This relates to standard A-CED.4, rearranging formulas.
Future Learning

In algebra 2, students will use the attributes of linear and quadratic relationships in real-world applications. Additionally, students will apply this knowledge to the translation of logarithmic and exponential relationships.

Additional Findings

In relation to quadratics John Allen Paulos wrote in *Beyond Numeracy*, “Many situations in physics, engineering, and elsewhere lead to such equations.” (p. 198)

*PARCC Model Content Frameworks for Mathematics* notes that “fluency in transforming expressions and chunking (seeing parts of an expression as a single object) is essential in factoring, completing the square and other mindful calculations.” (p. 52)
Algebra 1, Quarter 4, Unit 4.1

Solving Quadratic Equations Using Multiple Methods and
Solving Systems of Linear and Quadratic Equations (13 days)
Overview

Number of instructional days: 5 (1 day = 45 minutes)

Content to be learned

• Interpret key features of graphs and tables for linear, quadratic, and exponential functions.
• Identify attributes, including domain and range, of graphs of linear, quadratic, and exponential functions.
• Explain the meaning of the closure property for rational and irrational numbers.
• Calculate and interpret average rate of change of a function.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.
• Compare and contrast two separate arguments.
• Reason inductively.

Use appropriate tools strategically.
• Consider available tools to use in problem solving.
• Make sound decisions about when these tools might be helpful.
• Use technological tools to explore and deepen understanding.

Essential questions

• How does the domain of a function relate to its graph?
• What are the key features of graphs of linear, quadratic, and exponential functions?
• What does it mean for a set of numbers to be closed?
Written Curriculum

Common Core State Standards for Mathematical Content

Interpreting Functions

Interpret functions that arise in applications in terms of the context [Linear, exponential, and quadratic]

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

The Real Number System

Use properties of rational and irrational numbers.

N-RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

In unit 4.1, students learned how to graph and find solutions for linear and quadratic relationships.

Current Learning

Students develop an understanding of exponential relationships, comparing and contrasting key features of linear, quadratic, and exponential graphs.

Future Learning

In algebra 2, students will relate key features of the graphs of linear, quadratic, and exponential relationships to other polynomial relationships (cubic, quartic, etc.)

Additional Findings

Principles and Standards for School Mathematics notes that high school students should be able to interpret functions in a variety of formats. “Students should solve problems in which they use tables, graphs, words and symbolic expressions to represent and examine functions and patterns of change. Students should learn algebra both as a set of concepts and competencies tied to the representation of quantitative relationships and as a style of mathematical thinking for formalizing patterns, functions, and generalizations.” (p. 287)
Algebra 1, Quarter 4, Unit 4.3
Comparing, Graphing, Interpreting, and Classifying Functions Based on Their Properties

Overview

Number of instructional days: 13 (1 day = 45 minutes)

Content to be learned

- Graph linear, quadratic, square root, cube root, piecewise, step, absolute value, exponential, logarithmic, and trigonometric functions.
- Use the properties of exponents to interpret expressions for exponential functions.
- Use factoring and completing the square to show zeros, extreme values, and symmetry of the graph.
- Compare properties of two functions represented in different ways.

Essential questions

- How do the graphs of linear, quadratic, square root, cube root, piecewise, step, absolute value, exponential, logarithmic, and trigonometric functions compare?
- How does the format of the equation provide information about its graph’s characteristics?
- How can rewriting a function assist in finding certain characteristics of the function?
- How can factoring and completing the square be used to show zeros, extreme values, and symmetry of the graph of functions?

Mathematical practices to be integrated

- Reason abstractly and quantitatively.
- Make sense of quantities and relationships within a problem.
- Create representations of a problem.
- Use appropriate tools strategically.
- Use technology to solve problems, check solutions, visualize results, and compare predictions.
### Written Curriculum

**Common Core State Standards for Mathematical Content**

#### Interpreting Functions

<table>
<thead>
<tr>
<th>Analyze functions using different representations [Linear, exponential, quadratic, absolute value, step, piecewise-defined]</th>
<th>F-IF</th>
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<tbody>
<tr>
<td><strong>F-IF.7</strong> Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*</td>
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<tr>
<td>a. Graph linear and quadratic functions and show intercepts, maxima, and minima.</td>
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<tr>
<td>b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</td>
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<tr>
<td>e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</td>
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<tr>
<td><strong>F-IF.8</strong> Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</td>
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<tr>
<td>a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</td>
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<tr>
<td>b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as ( y = (1.02)^t ), ( y = (0.97)^t ), ( y = (1.01)^{12t} ), ( y = (1.2)^{t/10} ), and classify them as representing exponential growth or decay.</td>
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<tr>
<td><strong>F-IF.9</strong> Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</td>
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</tbody>
</table>

#### Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

In units 4.1 and 4.2, students graphed and interpreted linear and quadratic functions.

Current Learning

Students learn to graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Additionally, students begin to develop the skills needed to graph exponential, logarithmic, and trigonometric functions. Students learn the appropriate method to use to find specific characteristics/attributes of these functions (e.g., Use completing the square to transform a quadratic equation into Vertex Form to find the maximum or minimum value of the function.). Students compare the properties of two functions each represented in different ways.

Consider the comparison of the parent graphs of different types of functions to a transformed version of $x + 3$.

Focus on horizontal/vertical translations of functions.

Future Learning

In algebra 2 and precalculus, students will master graphing and interpreting the graphs of quadratic, polynomial, and trigonometric functions.

Additional Findings

Principles and Standards for School Mathematics notes

“High school students’ algebra experience should enable them to create and use tabular, symbolic, graphical, and verbal representations and to analyze and understand patterns, relations, and functions with more sophistication than in the middle grades.” (p. 297)

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“High school students should have substantial experience in exploring the properties of different classes of functions. For instance, they should learn that the function $f(x) = x^2 - 2x - 3$ is quadratic, that its graph is a parabola, and that the graph opens “up” because the leading coefficient is positive.

They should also learn that some quadratic equations do not have real roots and that this characteristic corresponds to the fact that their graphs do not cross the $x$-axis. Students should also be able to identify the complex roots of such quadratics.” (p. 299)
Algebra 1, Quarter 4, Unit 4.4
Comparing, Performing Operations, and Finding Inverses of Functions

Overview

Number of instructional days: 4 (1 day = 45 minutes)

Content to be learned

- Interpret expressions in terms of their context.
- Rewrite/rearrange exponential and inverse functions in a different form to identify key features of the function.
- Create equations or inequalities in one or more variables and graph the solution.
- Use technology to graph exponential, logarithmic, and inverse relationships to find solutions.
- Construct exponential functions using arithmetic (+/−) and geometric (×/÷) sequences.
- Identify exponential functions as increasing at the greatest rate of change.
- Identify translations of exponential functions.
- Distinguish between situations that can be modeled with linear functions and with exponential functions.
- Write an expression for the inverse of a given function.

Mathematical practices to be integrated

- Use appropriate tools strategically.
- Use technology to solve problems, check solutions, visualize results, and compare predictions.
- Use technological tools to explore and deepen understanding.

Essential questions

- How can you identify an exponential function from its graph and its equation?
- What patterns in data would distinguish it as linear, quadratic, exponential, or absolute value?
- What are the steps used to find the inverse of an exponential function?
- Where can you find examples of exponential relationships in real life?
### Written Curriculum

**Common Core State Standards for Mathematical Content**

**Linear, Quadratic, and Exponential Models**

<table>
<thead>
<tr>
<th>Construct and compare linear, quadratic, and exponential models and solve problems</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>F-LE.1</strong> Distinguish between situations that can be modeled with linear functions and with exponential functions.*</td>
</tr>
<tr>
<td>a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.</td>
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<tr>
<td>b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.</td>
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<tr>
<td>c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.</td>
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</tbody>
</table>

| **F-LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). |

| **F-LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.* |

<table>
<thead>
<tr>
<th>Building Functions</th>
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</thead>
<tbody>
<tr>
<td><strong>F-BF</strong></td>
</tr>
</tbody>
</table>

**Build a function that models a relationship between two quantities** *For F.BF.1, 2, linear, exponential, and quadratic*

| **F-BF.1** Write a function that describes a relationship between two quantities.* |
| a. Determine an explicit expression, a recursive process, or steps for calculation from a context. |
| b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* |

| **F-BF.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.* |

**Build new functions from existing functions** *Linear, exponential, quadratic, and absolute value; for F.BF.4a, linear only*

| **F-BF.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* |
F-BF.4 Find inverse functions
   a. Solve an equation of the form \( f(x) = c \) for a simple function \( f \) that has an inverse and write an expression for the inverse. *For example, \( f(x) = 2x^3 \) or \( f(x) = (x+1)/(x-1) \) for \( x \neq 1 \).*

### Seeing Structure in Expressions

**Interpret the structure of expressions**

<table>
<thead>
<tr>
<th>A-SSE</th>
<th>Linear, exponential, quadratic</th>
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</table>

**A-SSE.1** Interpret expressions that represent a quantity in terms of its context.*

   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret \( P(1+r)^n \) as the product of \( P \) and a factor not depending on \( P \).*

**A-SSE.2** Use the structure of an expression to identify ways to rewrite it. *For example, see \( x^4 - y^4 \) as \((x^2)^2 - (y^2)^2\), thus recognizing it as a difference of squares that can be factored as \((x^2 - y^2)(x^2 + y^2)\).*

### Write expressions in equivalent forms to solve problems

**A-SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*

   c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression \( 1.15^t \) can be rewritten as \((1.15^{1/12})^{12t} \approx 1.012^{12t} \) to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

### Create equations that describe numbers or relationships

**A-CED.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

**A-CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

**A-CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm’s law \( V = IR \) to highlight resistance \( R \).*

### Represent and solve equations and inequalities graphically

**A-REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

**A-REI.11** Explain why the \( x \)-coordinates of the points where the graphs of the equations \( y = f(x) \) and \( y = g(x) \) intersect are the solutions of the equation \( f(x) = g(x) \); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. *Include cases where \( f(x) \) and/or \( g(x) \) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

**A-REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
Understand the concept of a function and use function notation [Learn as general principle; focus on linear and exponential and on arithmetic and geometric sequences]

F-IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \).

F-IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by \( f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) \) for \( n \geq 1 \).

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function \( h(n) \) gives the number of person-hours it takes to assemble \( n \) engines in a factory, then the positive integers would be an appropriate domain for the function.*

F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

Common Core Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

Clarifying the Standards

Prior Learning

In unit 2.2, students constructed linear functions, and in unit 3.4, they constructed quadratic functions.

Current Learning

Students extend their knowledge of structure in expressions to include exponential and inverse functions. They examine graphs and data to determine that exponential functions increase more rapidly than any other functional relationship. Furthermore, students create simple rational and exponential functions.

Although there are numerous standards in this unit of study, teachers should focus on applying students’ prior knowledge of linear and quadratic relationships to exponential relationships.

Future Learning

In algebra 2, students will graph and solve exponential and logarithmic equations, and they will graph trigonometric functions. In precalculus, students will master graphing exponential and logarithmic functions and use the inverse of these functions to solve problems. Students will also graph trigonometric function in precalculus.

Additional Findings

None at this time.