SESSION 4A.1

Multiplying a Whole Number by a Fraction

Math Focus Points

- Using a representation to multiply a fraction and a whole number
- Extending understanding of the operation of multiplication to include fractions
- Writing multiplication equations for multiplying a fraction and a whole number

Today's Plan		Materials
1 ACTIVITY **The Big Bicycle Race**	40 MIN, CLASS, PAIRS	• *Student Activity Book*, pp. 71–72 or **C21–C22, Big Bicycle Race** Make copies. (as needed) • **C20, Fraction Bars** Make copies and a transparency.
2 DISCUSSION **Writing Equations**	20 MIN, CLASS	• *Student Activity Book*, p. 71 or C21 (completed)
3 SESSION FOLLOW-UP **Daily Practice**		• *Student Activity Book*, p. 73 or **C23, Take a Hike** Make copies. (as needed)

Ten-Minute Math

Estimation and Number Sense **Using Digit Cards (M5–M7), make two 5-digit plus 5-digit addition problems (_ _,_ _ _ + _ _,_ _ _). Give students 30 seconds to mentally estimate sums as close as possible to the exact answers. Students may write down partial sums if they wish. Some students may determine the exact answers. Have two or three students explain their work for each problem, and record these strategies on the board or overhead.**

ACTIVITY

1 The Big Bicycle Race

40 MIN CLASS PAIRS

Before the session begins, write the following information on the board:

The Big Bicycle Race is 480 miles long.

At the end of Day 1:
Nora has completed $\frac{1}{6}$ of the race.
Stuart has completed $\frac{1}{10}$ of the race.
Margaret has completed $\frac{1}{8}$ of the race.

We've been adding and subtracting fractions, and now we're going to multiply fractions. Think about this problem:

Nora, Stuart, and Margaret are long-distance bicycle racers. They enter long races that last several days. This year they are entering the Big Bicycle Race. The total distance they have to bike is 480 miles. Each day, they are allowed to race for 6 hours, and each evening is a rest period. The information on the board explains how far each person has cycled at the end of Day 1.

First, let's use what we already know about comparing fractions to think about who is winning the race at the end of Day 1. Who's in first place and who's in last place? How do you know? Turn and talk to a neighbor about this. Use what you know about comparing fractions.

Gather responses and discuss. Students should recognize that because $\frac{1}{6}$ is the largest fraction, Nora is winning at the end of Day 1. They should use similar reasoning to explain why Stuart is in last place. *($\frac{1}{10}$ is the smallest of the three fractions.)*

What if we wanted to know exactly how many miles each person has cycled? Let's start with Nora. If she has completed $\frac{1}{6}$ of the race, how many miles did she ride? How could we figure out $\frac{1}{6}$ of 480? Turn and talk to a neighbor about how you might solve this problem.

If students seem uncertain about how to answer this question, remind them that they solved similar problems in Investigations 1 and 2, such as *$\frac{1}{4}$ of a class of 32 students.* Stop students after a minute or so.

Some of you have already found the answer to this problem. Let's also think about a way to represent this problem.

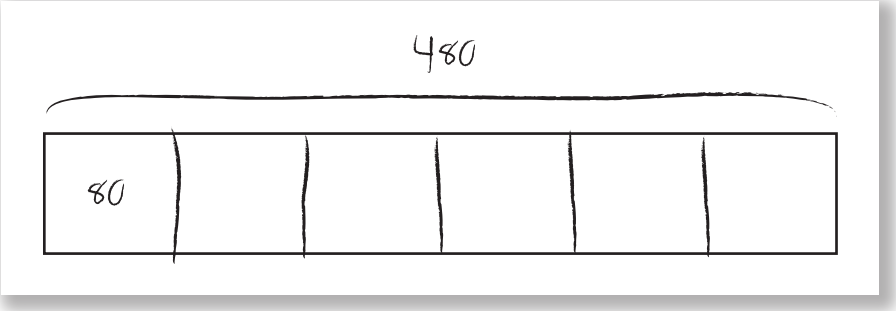
Distribute copies of Fraction Bars (C20) and display the transparency on the overhead. ❶

We're going to use these fraction bars during the next few sessions. The length of the bar represents the whole race—in this case, 480 miles. Work with a neighbor to show how far Nora has gone at the end of Day 1.

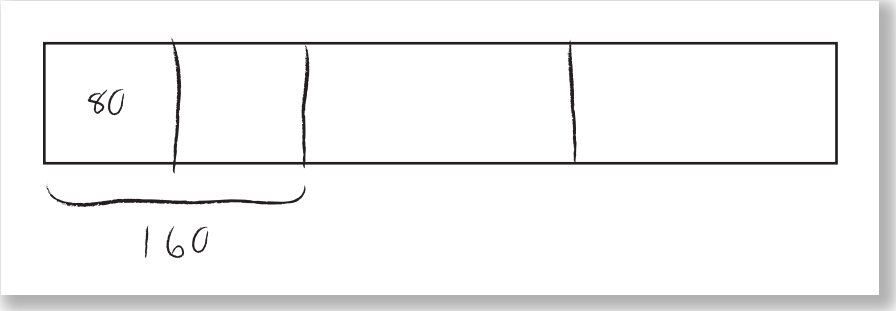
Give students a minute or two to work on this problem. Then ask students to share their solutions and explain their thinking.

Students might say:

"Since Nora had gone $\frac{1}{6}$ of the race, we divided the bar into 6 equal pieces. Since each piece shows the same amount, we divided 480 by 6 and that's 80. So $\frac{1}{6}$ of the race is 80 miles."

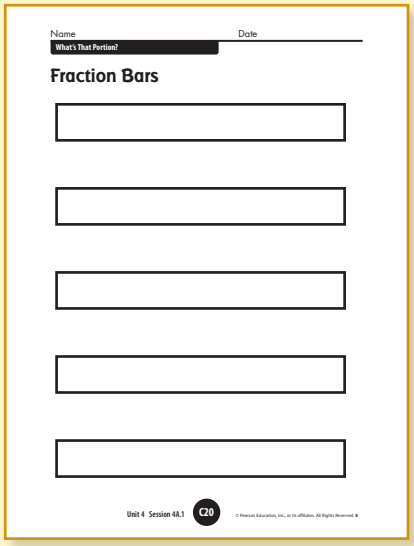


"First we divided the bar into thirds because that seemed an easy way to start, and we had to figure out how many miles that was. We divided 480 by 3 and that's 160. We know $\frac{1}{6}$ is a half of $\frac{1}{3}$, and half of 160 is 80. The answer is 80 miles."

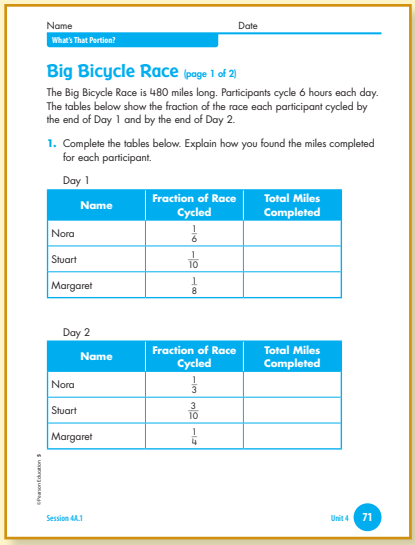


Teaching Note

❶ **Using the Fraction Bar** It is important in these first sessions of the Investigation to establish and use the fraction bar as a representation. Some students may want to use a number line to solve these problems. Acknowledge that the number line is a representation that works to find the solutions to these problems, and ask students to also use the fraction bar. The fraction bar is used in Sessions 4A.4–4A.5 to represent multiplying a fraction by a fraction, and it is important students have experience with the fraction bar before those sessions.


Name Date
What's That Portion?
Fraction Bars
Unit 4 Session 4A.1 C20

▲ Resource Masters, C20



Name Date

What's That Portion?

Big Bicycle Race (page 1 of 2)

The Big Bicycle Race is 480 miles long. Participants cycle 6 hours each day. The tables below show the fraction of the race each participant cycled by the end of Day 1 and by the end of Day 2.

1. Complete the tables below. Explain how you found the miles completed for each participant.

Day 1

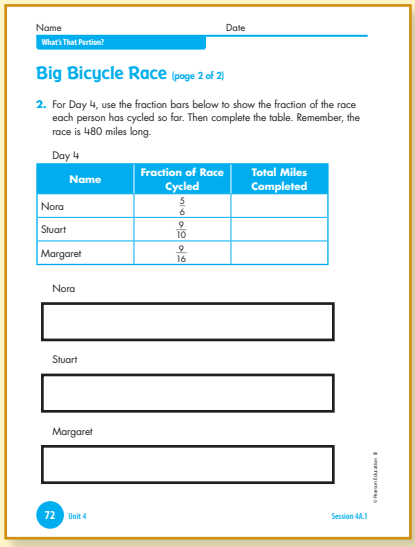
Name	Fraction of Race Cycled	Total Miles Completed
Nora	$\frac{1}{6}$	
Stuart	$\frac{1}{10}$	
Margaret	$\frac{1}{8}$	

Day 2

Name	Fraction of Race Cycled	Total Miles Completed
Nora	$\frac{1}{3}$	
Stuart	$\frac{3}{10}$	
Margaret	$\frac{1}{4}$	

Session 4A.1 Unit 4 71

▲ **Student Activity Book, Unit 4, p. 71; Resource Masters, C21**



Name Date

What's That Portion?

Big Bicycle Race (page 2 of 2)

2. For Day 4, use the fraction bars below to show the fraction of the race each person has cycled so far. Then complete the table. Remember, the race is 480 miles long.

Day 4

Name	Fraction of Race Cycled	Total Miles Completed
Nora	$\frac{5}{6}$	
Stuart	$\frac{9}{10}$	
Margaret	$\frac{9}{16}$	

Nora

Stuart

Margaret

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▲ **Student Activity Book, Unit 4, p. 72; Resource Masters, C22**

Record one solution on the transparency, and save the transparency for the discussion at the end of the session.

Ask students to look at *Student Activity Book* pages 71–72 or C21–C22 and see what they notice. Point out that there is no table for Day 3, and that all students are expected to use the fraction-bar representation for Day 4. Students can also use copies of C20 for completing the other tables.

ONGOING ASSESSMENT: Observing Students at Work

Students solve multiplication problems involving fractions and whole numbers.

- **How are students solving the problems?** Are they using the fraction bars to help them solve the problems? (e.g., To find $\frac{1}{8}$ for Margaret for Day 1, are they dividing the bar into $\frac{1}{2}$ and showing 240 miles, $\frac{1}{4}$ to show 120 miles, and $\frac{1}{8}$ to show 60 miles?) Are they dividing 480 by the denominator to determine the total miles completed?
- **Are students noticing the relationship between the problems?** (e.g., For Day 2, do they think about $\frac{1}{3}$ or $\frac{1}{4}$ as being twice as far as $\frac{1}{6}$ or $\frac{1}{8}$? Or that $\frac{3}{10}$ is 3 times as far as $\frac{1}{10}$?)
- **How are students dividing the fraction bar for Day 4?** Do students understand that the fraction bars should be divided into equal parts? Are students using the relationship between sixths and thirds; between fourths, eighths, and sixteenths?

DIFFERENTIATION: Supporting the Range of Learners

Intervention Some students may not know how to approach solving these problems. Ask these students to only work on Day 1 and to use the fraction bars to help them solve the problem.

Extension For students who quickly solve all of the problems, ask them to estimate the fraction of the race each participant had cycled on Day 3.

DISCUSSION

2 Writing Equations

20 MIN CLASS

Math Focus Points for Discussion

- Writing multiplication equations for multiplying a fraction and a whole number

Display the transparency showing $\frac{1}{6}$ of 480 is 80.

At the end of Day 1, Nora had completed $\frac{1}{6}$ of the race. You figured out that was 80 miles.

Write on board: $\frac{1}{6}$ of 480 miles is 80 miles.

How would you write this as an equation? Talk to a neighbor and come up with an equation.

Gather students' ideas. It is likely a student will suggest the equation $480 \div 6 = 80$. Acknowledge that this equation correctly represents the situation.

But what if you use the numbers $\frac{1}{6}$, 480, and 80? Those are the numbers used on *Student Activity Book* page 71 or C21. How can you write an equation using these three numbers to represent the situation?

Record suggestions. Take a few minutes for students to explain their ideas. It is likely that not all students will understand why this problem is a multiplication situation.

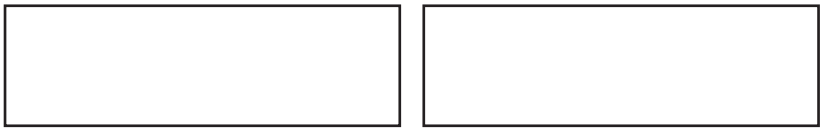
Let's look at a bike race with whole numbers and then come back to fractions.

Draw a rectangular bar on the board, and explain that this time, the race is 100 miles long.

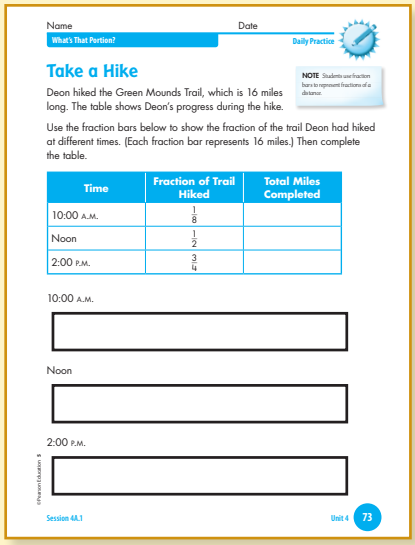
100 miles

This bar represents a race of 100 miles. What if Olivia biked 2 of these races? How would you show that?

Draw two bars:



100 miles 100 miles



Name Date

What's That Portion? Daily Practice

Take a Hike

Deon hiked the Green Mounds Trail, which is 16 miles long. The table shows Deon's progress during the hike.

NOTE Students use fraction bars to represent fractions of a distance.

Use the fraction bars below to show the fraction of the trail Deon had hiked at different times. (Each fraction bar represents 16 miles.) Then complete the table.

Time	Fraction of Trail Hiked	Total Miles Completed
10:00 A.M.	$\frac{1}{8}$	
Noon	$\frac{1}{2}$	
2:00 P.M.	$\frac{3}{4}$	

10:00 A.M.

Noon

2:00 P.M.

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▲ **Student Activity Book, Unit 4, p. 73; Resource Masters, C23**

How would you write an equation to show the total distance Olivia biked in two races?

Record $2 \times 100 = 200$.

Where is the 2 in this representation? The 100? The 200? What if she biked one of these races? What would the picture look like? What would the equation be?

Record $1 \times 100 = 100$.

What if Olivia's bike broke down, and she could only bike half of the 100-mile race? What would the picture look like? What equation would I write?

Refer students back to the whole-number equations as necessary.

For 2 races of 100 miles each, we write 2 times 100. For 1 race of 100 miles, we write 1 times 100. For $\frac{1}{2}$ of a race of 100 miles, what could we write?

Record $\frac{1}{2} \times 100 = 50$.

Tell students that when using fractions, a fraction *of* something indicates multiplication. Just like 2 groups *of* something is multiplication, $\frac{1}{2}$ group *of* something is also multiplication.

Display the transparency, again, showing $\frac{1}{6}$ of 480. Write $\frac{1}{6} \times 480 = 80$ underneath the fraction bar.

Let's look at this solution one more time. Where do you see 480 in the representation? $\frac{1}{6}$? 80?

In the next few sessions we're going to continue working on multiplying whole numbers, fractions, and mixed numbers.

3 SESSION FOLLOW-UP
Daily Practice

Daily Practice: For reinforcement of this unit's content, have students complete *Student Activity Book* page 73 or C23.

Name Date

What's That Portion?

Fraction Bars

Big Bicycle Race (page 1 of 2)

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Margaret	$\frac{1}{8}$	

Day 2

Name	Fraction of Race Cycled	Total Miles Completed
Nora	$\frac{1}{3}$	
Stuart	$\frac{3}{10}$	
Margaret	$\frac{1}{4}$	

Big Bicycle Race (page 2 of 2)

2. For Day 4, use the fraction bars below to show the fraction of the race each person has cycled so far. Then complete the table. Remember, the race is 480 miles long.

Day 4

Name	Fraction of Race Cycled	Total Miles Completed
Nora	$\frac{5}{6}$	
Stuart	$\frac{9}{10}$	
Margaret	$\frac{9}{16}$	

Nora

Stuart

Margaret

Take a Hike

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Noon	$\frac{1}{2}$	
2:00 P.M.	$\frac{3}{4}$	

10:00 A.M.

Noon

2:00 P.M.