Kindergarten Mathematics, Quarter 1, Unit 1.1

Counting and Cardinality—Counting to 10 and Conceptual Understanding to 5

Overview

Number of instructional days: 10 (1 day = 30 minutes)

Content to be learned

- Rote count to 10.
- Write numbers up to 5.
- Represent numbers with objects up to 5.
- Represent numbers of objects with numerals up to 5.
- Understand one-to-one correspondence up to 5.
- Recognize numerals to 5.
- Understand cardinality of numbers.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Explain to themselves the meaning of a problem.
- Look for an entry point to a problem's solution.
- Use concrete objects or pictures to help conceptualize and solve a problem.
- Question solutions; monitor and adjust path and ask, "Does this make sense?"

Construct viable arguments and critique the reasoning of others.

- Listen to arguments, asking, "Does this make sense?"
- Ask useful questions to clarify and improve arguments.
- Analyze situations and justify conclusions through verbal explanations.

Essential questions

- How can you count these objects?
- Why is it important to give a number name to each object?
- How can you represent the number of objects you counted?
- How do you know how many objects you counted?

Written Curriculum

Common Core State Standards for Mathematical Content

Counting and Cardinality	**K.CC**

Know number names and the count sequence.

K.CC.1 Count ~~to 100~~ by ones ~~and by tens~~.

K.CC.3 Write numbers ~~from 0 to 20~~. Represent a number of objects with a written numeral ~~0–20 (with 0 representing a count of no objects)~~.

Count to tell the number of objects.

K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.

a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings,

diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Clarifying the Standards

Prior Learning

Students came to school with limited and varied educational experiences (i.e., Head Start, a preschool setting, within the home environment, or no experience at all). Students entered with a range of mathematical understanding—for example, they may have been able to rote count, but not have been able to attach meaning. Whereas, others may have been able to attach a name and symbol and represent the number given.

Current Learning

Students learn the number names and count in sequential order up to 10. They represent a number of objects with a written number to 5 and match one-to-one correspondence. Students continue to build their number sense to 100 over the course of the year. They use this knowledge to count with cardinality; they understand greater than/less than, perform addition and subtraction, and count from different starting points.

Future Learning

In grade 1, students will relate counting to addition and subtraction. They will count to 120 from any number less than 120. Students will read and write numerals and represent a number of objects with a written numeral.

Additional Findings

According to *Curriculum Focal Points*, "Children use numbers, including written numerals, to represent quantities … such as counting objects in a set, creating a set with a given number of objects … ordering sets or numerals by using both cardinal and ordinal meanings." (p. 12)

According to *Principles and Standards for School Mathematics*, students will understand numbers, ways of representing numbers, and the number system. (p. 32)

Kindergarten Mathematics, Quarter 1, Unit 1.2

Counting and Cardinality—Counting to 20 and Conceptual Understanding to 10

Overview

Number of instructional days: 10 (1 day = 30 minutes)

Content to be learned

- Rote count to 20.
- Write numbers up to 10.
- Represent numbers with objects up to 10.
- Represent numbers of objects with numerals up to 10.
- Understand one-to-one correspondence up to 10.
- Recognize numerals to 10.
- Understand cardinality of numbers.

Mathematical practices to be integrated

Attend to precision.

- Communicate their understanding of quantities precisely to others.
- Use clear definitions when describing quantities to themselves and others.

Look for and make sense of structure.

- Look at the relationship between number and quantities.
- Discover patterns in the structure of the number system.

Essential questions

- How do you show and count numbers?
- How many ways can you show the quantity of 10?
- How can you represent the amount you counted?
- How do you know what number comes next when you are counting?
- How can you match the objects with the numerals?

Written Curriculum

Common Core State Standards for Mathematical Content

Counting and Cardinality	K.CC

Know number names and the count sequence.

K.CC.1 Count ~~to 100~~ by ones ~~and by tens~~.

K.CC.3 Write numbers ~~from 0 to 20~~. Represent a number of objects with a written numeral ~~0–20 (with 0 representing a count of no objects)~~.

Count to tell the number of objects.

K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.

a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.

Common Core Standards for Mathematical Practice

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

Students came to school with limited and varied educational experiences (i.e., Head Start, a preschool setting, within the home environment, or no experience at all). Students entered with a range of mathematical understanding—for example, they may have been able to rote count, but not have been able to attach meaning. Whereas, others may have been able to attach a name and symbol and represent the number given.

Current Learning

Students learn the number names and count in sequential order up to 20. They represent a number of objects with a written number to 10 and match one-to-one correspondence. Students continue to build their number sense to 100 over the course of the year. They use this knowledge to count with cardinality; they understand greater than/less than, perform addition and subtraction, and count from different starting points.

Future Learning

In grade 1, students will relate counting to addition and subtraction. They will count to 120 from any number less than 120. They will read and write numerals and represent a number of objects with a written numeral.

Additional Findings

According to *A Research Companion to Principles and Standards for School Mathematics*, "Several types of evidence indicate that children understand all [5] counting principals by age 5." (p. 290)

The counting principals are as follows:

1. **One-One Principle:** One and only one number is assigned to each object.
2. **Stable Order Principle:** The numbers are always assigned in the same order.
3. **Cardinal Principle:** The last count indicates the number of objects in the set.
4. **Order Irrelevance Principle:** The order in which objects are counted is irrelevant.
5. **Abstraction Principle:** The other principles apply to any set of objects.

Kindergarten Mathematics, Quarter 1, Unit 1.3

How Many in a Set? Numbers to 10

Overview

Number of instructional days: 10 (1 day = 30 minutes)

Content to be learned

- Count up to 10 objects in a line or in other configurations.
- Given a number from 1 to 10, count out that many objects.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Explain to themselves the meaning of a problem.
- Look for an entry point to a problem's solution.
- Use concrete objects or pictures to help conceptualize and solve a problem.
- Question solutions, monitor and adjust path, and ask, "Does this make sense?"

Essential questions

- How do you decide how many objects are in a group?
- How can you tell if you have counted them all?

Written Curriculum

Common Core State Standards for Mathematical Content

Counting and Cardinality	K.CC

Count to tell the number of objects.

K.CC.5 Count to answer "how many?" questions about ~~as many as 20~~ things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number ~~from 1–20~~, count out that many objects.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Clarifying the Standards

Prior Learning

Students may have had experience counting objects at home or preschool. They heard number names said and may have attached meaning to the number names. Students' experiences were varied and depended on the environments in which they developed.

Current Learning

Students represent a number of objects with a written number to 10 and match one-to-one correspondence. They answer the question "How many in a set?" and know that the last number said tells the number of objects. Later in the year, students count as many as 20 objects arranged in a line, rectangular array, or circle. Students continue to build their number sense (the ability for students to say, write, identify, and represent numbers) over the course of the year. They use this knowledge to count with cardinality; they understand that one number is greater than or less than another number. To perform addition and subtraction, students count on from different starting points.

Future Learning

In grade 1, students will relate counting to addition and subtraction. They will count to 120 from any number less than 120. They will demonstrate proficiency with addition and subtraction within 10. Students will use addition and subtraction within 20 to solve word problems using objects, drawings, and equations with a symbol for the unknown number to represent the problem. They will read and write numerals and represent a number of objects with a written numeral.

Additional Findings

According to *Principles and Standards for School Mathematics*, "Young children's earliest mathematical reasoning is likely to be about number situations, and their first mathematical representations will probably be of numbers. Research has shown that learning about number and operations is a complex process for children." (p. 32)

According to *A Research Companion to Principles and Standards for School Mathematics*, "At 3 or 4 years of age, children become proficient in another means of establishing the cardinal value of a set-*counting*." (p. 290) It further states that many children recognize that counting can start in the middle of a row of objects, as long as each object is counted.

Kindergarten Mathematics, Quarter 1, Unit 1.4

Two-Dimensional Shapes

Overview

Number of instructional days: 10 (1 day = 30 minutes)

Content to be learned

- Identify and describe shapes (squares, circles, triangles, rectangles, hexagons).
- Describe objects in the environment using names of shapes (squares, circles, triangles, rectangles, hexagons).
- Correctly names shapes (squares, circles, triangles, rectangles, hexagons) regardless of their overall size.
- Construct shapes (squares, circles, triangles, rectangles, hexagons) from objects (e.g., sticks and clay balls).

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Make verbal statements and explain their reasoning using concrete objects and prior knowledge.
- Make sense of others thinking based on their previous experience and ask useful questions to clarify.

Attend to precision.

- Communicate clearly to others.
- Use clear definitions to demonstrate their reasoning.

Essential questions

- How are a square and a rectangle similar? How are they different?
- How can you identify a shape? (squares, circles, triangles, rectangles, hexagons)
- How can you construct a hexagon using objects?
- Where can you find something that has the same shape as a _____ (square, circle, triangle, rectangle, hexagon)?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry	**K.G**

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

K.G.1 Describe objects in the environment using names of shapes~~, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*~~.

K.G.2 Correctly name shapes regardless of ~~their orientations or~~ overall size.

Analyze, compare, create, and compose shapes.

K.G.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) ~~and drawing shapes~~.

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

Students came to school with limited and varied educational experiences (i.e., Head Start, a preschool setting, within the home environment, or no experience at all). Students entered with a range of mathematical understanding in geometry—for example, they may have been able to identify some shapes, but not others. They may have had misconceptions about shapes (e.g., a hexagon is an octagon; not knowing the similarities and differences between a square and rectangle). Whereas, other students may have been able to attach a name to a shape, define that shape, and find it in the environment.

Current Learning

Students identify and describe two-dimensional shapes and find them in the environment regardless of their overall size. Later, students are able to identify and describe two- and three-dimensional objects in the environment regardless of their orientation or size using positional terms such as *above, below, beside, in front of,* and *next to*. They further define two-dimensional shapes as flat (lying in a plane) and three-dimensional shapes as a solid.

Future Learning

Students will distinguish between defining (e.g., triangles are closed and three sided) and nondefining (e.g., color, orientation, overall size) attributes. They will use these attributes to build and draw shapes. Students will compose two- or three-dimensional shapes to create a composite shape. They will partition circles and rectangles into two and four equal shares and describe the shares using halves and quarters. Students will understand that decomposing into more equal shares creates smaller shares.

Additional Findings

According to *Curriculum Focal Points*, "Children interpret the physical world with geometric ideas (e.g., shape, orientation, spatial relations) and describe it with corresponding vocabulary. They identify, name, and describe a variety of shapes, such as squares, triangle, circles, rectangles, (regular) hexagons, and (isosceles) trapezoids presented in a variety of ways (e.g., with different sizes and orientations), as well as such three-dimensional shapes as spheres, cubes, and cylinders. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes." (p. 12)

According to *Principles and Standards for School Mathematics*, "Piaget and Inhelder's first theme—that children's representation of space is constructed from active manipulation of their spatial environment—has been supported (Clements & Battista, 1992). Children's ideas about shapes do not come from passive looking. Instead, they come from children's bodies, hands, eyes, and minds engaged in active construction. In addition, children need to explore shapes extensively to fully understand them; merely seen in naming pictures is insufficient. Finally, they have to explore the parts and attributes of shapes." (p. 152)

The following are taken from *Principles and Standards for School Mathematics* (pp. 152–153):

- Level 0—level at which children do not reliably distinguish circles, triangles, and squares from nonexemplars from those classes and appear unable to form reliable mental images from these shapes.
- Level 1—the visual level, at which students can recognize shapes only has wholes and cannot form mental images of them.
- Level 2—the descriptive/analytical level, at which students recognize and categorize shapes by their properties.