Kindergarten Mathematics, Quarter 2, Unit 2.1

Counting to 40 and Understanding Numbers 0–20

Overview

Number of instructional days: 9 (1 day = 30 minutes)

Content to be learned

- Rote count to 40.
- Write numbers to 20.
- Represent numbers with objects to 20.
- Represent up to 20 objects with numerals.
- Understand that the number of objects counted is the last number said.

Mathematical practices to be integrated

Use appropriate tools strategically.

- Use manipulatives as counting tools.
- Select useful and efficient tools for the situation.

Look for and make use of structure.

- Look at the relationship between number and quantity.
- Discover and name patterns in the structure of the number system.

Essential questions

- How do you know how many objects are in the group?
- How can you represent this number with objects?
- How can you count aloud to 40?
- How can you represent these objects with a numeral?

Written Curriculum

Common Core State Standards for Mathematical Content

Counting and Cardinality	K.CC

Know number names and the count sequence.

K.CC.1 Count ~~to 100~~ by ones and by tens.

K.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

Count to tell the number of objects.

K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.

b. Understand that the last number name said tells the number of objects counted. ~~The number of objects is the same regardless of their arrangement or the order in which they were counted.~~

Common Core Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

Due to varied prekindergarten experiences, students entered school with a range of mathematical understanding. Some were able to attach a number name and/or a symbol and represent the number and some were not.

Current Learning

In the previous unit, students learned to rote count to 20, write numbers up to 10, and represent them with objects. In this unit, students learn that the last number said in a group of objects represents the number of objects. Students count by tens up to 40 (0, 10, 20, 30, and 40). They represent a number of objects with a written number to 20 and match one-to-one correspondence. Students are introduced to zero, with zero representing a count of no objects. By the end of kindergarten, students count to 100 by ones and tens.

Future Learning

In grade 1, students will continue to build their number sense to 100 over the course of the year. They will extend the counting sequence to 120. Numbers to 120 will be read, written, and represented with objects. Counting by ones and tens will build foundational skills for place value. Students will understand that the two digits of a two-digit number represent amounts of tens and ones.

Additional Findings

According to *Curriculum Focal Points*, "Children use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set, creating a set with a given number of objects …" (p. 24)

According to *Principals and Standards for School Mathematics,* students will "develop understanding of the relative position and magnitude of whole numbers and of ordinal and cardinal numbers and their connections." (p. 78)

The Research Companion to the PSSN, Chapter 6, p. 68—Developing Mathematical Power and Whole Number Operations, "Intertwining real world situations, problem solving, and computation for making sense and computational fluency."

Kindergarten Mathematics, Quarter 2, Unit 2.2

Counting and Classifying Objects

Overview

Number of instructional days: 9 (1 day = 30 minutes)

Content to be learned

- Count up to 20 objects in a line or in other configurations.
- Count up to 10 things in a scattered configuration.
- Count out given number of objects up to 20.
- Classify objects into given categories.
- Count the number of objects in each category.

Mathematical practices to be integrated

Make sense of problems and preserve in solving them.

- Think about categories given and what they mean.
- Count and recount objects in different configurations.
- Use concrete objects to count out a given number of objects.

Construct viable arguments and critique the reasoning of others.

- Ask useful questions to clarify why an object fits in a given category.
- Justify conclusions through verbal explanations of categories and objects.

Essential questions

- How do you know how many objects are in a group?
- How do you know if you counted them all?
- Why is it important to sort objects into categories?
- What group would this object go into and why?
- How do you know if your count is correct?
- How can you count _____(0–20) objects?

Written Curriculum

Common Core State Standards for Mathematical Content

Counting and Cardinality	K.CC

Count to tell the number of objects.

K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Measurement and Data	K.MD

Classify objects and count the number of objects in each category.

K.MD.3 Classify objects into given categories; count the numbers of objects in each category ~~and sort the categories by count~~.[3]

[3] Limit category counts to be less than or equal to 10.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Clarifying the Standards

Prior Learning

There was a wide range of mathematical understanding among the students in prekindergarten. Many children had experience counting small quantities. They often had sorted items by color or size.

Current Learning

In this unit, students learn how to answer the question, “How many?” by using up to 20 objects in various formations (e.g., a line, rectangular array, circle). When viewing scattered configurations, they name as many as 10 objects. When given a number 1–20, students count out that number of objects. They learn how to classify objects into given categories and to count the number of objects in each category. (Limit category counts to less than or equal to 10.)

Future Learning

In grade 1, students will count to 120 starting at any number less than 120. They also read and write numerals and represent a number of objects with a written numeral. Students develop a foundational understanding of place value of tens and ones.

Additional Findings

According to *Principles and Standards of School Mathematics,* in kindergarten, students “count with understanding and recognize ‘how many’ in sets of objects.” (p. 78) In addition, “Children should learn that the last number named represents the last object as well as the total number of objects in the collection.” (p. 79)

Kindergarten Mathematics, Quarter 2, Unit 2.3

Describing Shapes and Their Positions

Overview

Number of instructional days: 5 (1 day = 30 minutes)

Content to be learned

- Describe objects in the environment using names of shapes.
- Use positional terms to describe the location of a shape or object.
- Name shapes correctly regardless of their orientations or overall size.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Make verbal statements and explain their reasoning using concrete objects and prior knowledge.
- Ask questions to clarify the naming of shapes regardless of their sizes and orientations.

Attend to precision.

- Communicate relative location of objects.
- Use clear definitions to demonstrate their reasoning of objects in the environment.

Essential questions

- How can you sort these shapes? (Give students a group of attribute shapes.)
- What are three shapes in the environment that look like a ____?
- How can you show a shape (positional word) another shape?
- How can you describe the location of the object?
- How do you know this is a ___ (name a shape)?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry	K.G

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind*, and *next to*.

K.G.2 Correctly name shapes regardless of their orientations or overall size.

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

Students identified shapes in their environment due to their prior experiences. Some students were able to internalize positional words such as *above, below,* and *next to.*

Current Learning

In kindergarten, students learn the name of shapes of three-dimensional shapes and revisit two-dimensional shapes from Quarter 1. Students use terms such as *above, below, beside, in front of, behind,* and *next to* describe the position of objects. They understand that the orientation or overall size of a given shape does not change the shape itself.

Future Learning

In grade 1, students will define the attributes of the shape (triangles are closed and three-sided) and identify nondefined attributes such as color or size. They will also build and draw shapes by their attributes. Students will compose two-dimensional or three-dimensional shapes to create a composite shape and compose new shapes from the composite shape.

Additional Findings

According to *Curriculum Focal Points,* "Children interpret the physical world with geometric ideas and describe it with corresponding vocabulary. They identify, name, and describe a variety of shapes, such as squares, triangles, circles, rectangles, (regular) hexagons, and (isosceles) trapezoids presented in a variety of ways (e.g., with different sizes or orientations), as well as such 3-D shapes as spheres, cubes, and cylinders." (p. 12)

According to *Principles and Standards for School Mathematics,* "At first, young children learn concepts of relative position, such as above, behind, near, and between. (p. 42)

According to *Research Companion to the Principles and Standards for School Mathematics,* "Students learn that there is a progressive organization of geometric ideas that follows a definite order. They first learn topological relations (connectedness, continuity, enclosure) followed by projected relations (parallel lines, angle sizes, and other attributes). According to Dina van Hiele, students go through levels of thought in geometry. In level 1(where most kindergarten students are) students will recognize shapes only as wholes and cannot form mental images of them (e.g., a figure is a rectangle because it looks like a door)." (p. 152)

Kindergarten Mathematics, Quarter 2, Unit 2.4

Using Measureable Attributes to Describe Objects

Overview

Number of instructional days: 5 (1 day = 30 minutes)

Content to be learned

- Describe measurable attributes of objects (length, weight, etc.).

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Make verbal statements and explain their reasoning using concrete objects and prior knowledge.
- Make sense of others' thinking based on their previous experience and ask useful questions to clarify.

Attend to precision.

- Communicate clearly to others using mathematical vocabulary.
- Use clear explanations to demonstrate their reasoning.

Essential questions

- How can you describe this object?
- How do you know which object is longer (shorter, heavier, etc.)?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data	K.MD

Describe and compare measurable attributes.

K.MD.1 Describe measurable attributes of objects, such as length or weight. ~~Describe several measurable attributes of a single object.~~

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

Students entered with a range of mathematical understanding in measurement. In early childhood, students drew upon their experiences with objects standing for others, such as when they pretend played with a banana to represent a telephone because of its measurable length being similar to a telephone.

Current Learning

Students describe an object by measurable attributes such as length or weight. (Students distinguish between measurable and nonmeasurable attributes, such as color, in order to describe and compare attributes.) Later in the year, students describe several measurable attributes of a single object and directly compare two objects with a measurable attribute in common to see which object has "more of/ less of" the attribute and describe the difference.

Future Learning

In grade 1, students will order three objects by length. They will compare the lengths of two objects indirectly by using a third object. Students will express the length of an object as a whole number of unit lengths. They will understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.

In grade 2, students will use standard units of measure.

Additional Findings

A Research Companion to Principles and Standards for School Mathematics states, "Measurement has traditionally served as one route for developing mathematical knowledge of space (Kline, 1959). A pedagogical implication of this history is that learning to measure qualities of space, such as length, area, volume, and angle, provides a practical means for developing the fundamental understandings of its structure." (p. 180)

According to *A Research Companion to Principles and Standards for School Mathematics*, "Children's first understanding of length measure often involves direct comparison of objects (Lindquist, 1989; Piaget et al., 1960). Congruent objects have equal lengths, and congruency is readily tested when objects can be superimposed or juxtaposed." (p. 182)

Curriculum Focal Points states, "Children use measurable attributes, such as length or weight, to solve problems by comparing and ordering objects. They compare the lengths of two objects both directly (by comparing them with each other) and indirectly (by comparing both with a third object), and they order several objects according to length." (p. 12)

According to *Principles and Standards for School Mathematics*, "Children should begin to develop an understanding of attributes by looking at, touching, or directly comparing objects." (p. 103)

Kindergarten Mathematics, Quarter 2, Unit 2.5

Representing Addition Problems

Overview

Number of instructional days: 12 (1 day = 30 minutes)

Content to be learned

- Represent addition problems using drawings, sounds, mental images, objects, and fingers.
- Solve addition word problems within 10.
- Use objects or drawings to represent an addition problem.

Mathematical practices to be integrated

Make sense of problems and preserve in solving them.

- Explain to themselves the meaning of a problem.
- Look for an entry point to a problem's solution.
- Use concrete objects or pictures to help conceptualize and solve an addition problem.

Reason abstractly and quantitatively.

- Make sense of quantities and their relationship in problem situations.
- Make a representation of the problem.

Essential questions

- How many different ways can you solve this problem? (e.g., 4 and 2)
- How many different ways can you represent this problem ___? (e.g., 5)

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	**K.OA**

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1 Represent addition ~~and subtraction~~ with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations~~, expressions, or equations~~.

[2] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

K.OA.2 Solve addition ~~and subtraction~~ word problems, and add ~~and subtract~~ within 10, e.g., by using objects or drawings to represent the problem.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Clarifying the Standards

Prior Learning

Students entered kindergarten with a range of experiences in mathematics. Some students knew the concept of *more*, meaning the quantity gets larger. Most students had a better understanding of the term *more* than the term *less*.

Current Learning

Students develop understanding of the concept of addition within 10 by acting out situations, drawings, sounds, verbal explanations, and other methods. Students solve addition word problems by using objects and drawings. By the end of the year, they use expressions or equations to represent addition and subtraction situations.

Future Learning

In grade 1, students will use addition and subtraction to solve word problems within 20. The word problems will involve adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions. Drawings and objects will still be used to solve problems in addition to using the symbols to write equations and a symbol for the unknown. Eventually, students will solve word problems that involve adding three whole numbers with a sum that is less than or equal to 20.

Additional Findings

According to *Curriculum Focal Points,* "Children use meanings of numbers to create strategies for solving problems and responding to practical situations, such as getting just enough napkins for the group." (p. 24)

According to *Principles and Standards for School Mathematics,* "In pre-k to grade 2 all students should understand various meanings of addition and subtraction of whole numbers and the relationship between the two operations." (p. 78)