Kindergarten Mathematics, Quarter 4, Unit 4.1

Orally Counting to 100 by Ones and Tens

Overview

Number of instructional days: 5 (1 day = 30 minutes)

Content to be learned

- Rote count to 100 by ones.
- Count to 100 by tens.
- Count forward beginning at any number.

Mathematical practices to be integrated

Look for and make use of structure.

- Discover patterns in the structure of the number system.
- Understand the pattern of counting by tens.

Look for and express regularity in repeated reasoning.

- Look for and extend patterns.
- Look for regularity in repeating numbers.
- Use known patterns to extend the counting sequence.

Essential questions

- How do you know what number comes next when you are counting?
- How do you count to 100?
- How can you count by tens?
- What number patterns do you know?

Written Curriculum

Counting and Cardinality	**K.CC**

Know number names and the count sequence.

K.CC.1 Count to 100 by ones and by tens.

K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

Common Core Standards for Mathematical Practice

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

Students entered kindergarten with a range of mathematical understanding. For example, they may have been able to rote count, but may not have been able to attach meaning. Whereas, others may have been able to attach a name, symbol, and representation to a given number. Children have exposure to understanding whole numbers and number words referring to quantity. Some students are developing an understanding of one-to-one correspondence.

Current Learning

In quarter 3, students learn to count up to 70 by ones and tens. They also learn to count in sequential order starting at any given number. In this unit, students are learning to extend the counting sequence to 100 by ones and tens. They count to 100 from any given number without beginning at 1.

Future Learning

In grade 1, students will extend the counting sequence to 120. They will start at any number less than 120. Students will be able to read and write numerals and represent a number of objects with a written numeral.

Additional Findings

According to *Curriculum Focal Points* states, "Children use numbers, including written numerals, to represent quantities and to solve quantitative problems." (p. 12)

According to *Principles and Standards for School Mathematics, "*The Numbers and Operations Standard describes deep and fundamental understanding of, and proficiency with, counting, numbers, and arithmetic, as well as an understanding of number systems and their structures." (p. 32)

Kindergarten Mathematics, Quarter 4, Unit 4.2

Comparing Numbers

Overview

Number of instructional days: 5 (1 day = 30 minutes)

Content to be learned

- Identify the number of objects in one group as greater than or less than.
- Identify one group of (up to 10) objects as equal to another group of objects.
- Use matching and counting strategies to determine less than, greater than, or equal to.
- Compare two written numerals between 1 and 10.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Explain the correspondence between two groups of objects.
- Analyze what is given and look for relationships.
- Use concrete objects or pictures to help conceptualize and solve a problem.
- Question solutions; monitor and adjust path and ask, "Does this make sense?"

Model with mathematics.

- Students apply the counting strategies they know to compare quantities.
- Represent a situation with objects or numbers.
- Analyze relationships mathematically to draw conclusions.

Essential questions

- How do you know if the number of objects in this group is less than/greater than the number of objects in another group?
- How do you know if two groups are equal?
- How can you compare two numerals? Which is less? How do you know?

Written Curriculum

Counting and Cardinality	K.CC

Compare numbers.

K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1]

[1] Include groups with up to ten objects.

K.CC.7 Compare two numbers between 1 and 10 presented as written numerals.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

Students came to school with a wide variety of mathematical abilities. Some will be able to recognize "more or less than" pertaining to a group of objects. Other students may be able to count a set of objects. They understand that the last number in the set tells how many. They can compare quantities of two numerals.

Current Learning

Students identify the number of objects in a group as "greater than, less than, and/or equal to." They count a set of objects within a group of 10. They compare two numbers when written as numerals.

Future Learning

In first grade, students will compare two-digit numbers and record the comparisons using >, =, and/or < symbols to represent greater than, equal to, and less than.

Additional Findings

Curriculum Focal Points states, "Children use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set, creating a set with a given number of objects, comparing and ordering sets or numerals by using both cardinal and ordinal meanings, and modeling simple joining and separating situations with objects." (p. 12)

Kindergarten Mathematics, Quarter 4, Unit 4.3

Composing/Decomposing Number Sentences and Fluency with Facts to 5

Overview

Number of instructional days: 15 (1 day = 30 minutes)

Content to be learned

- Decompose numbers less than or equal to 10.
- Recognize a number sentence using a pair of numbers to express a decomposition (e.g., 5 = 2 + 3 or 5 = 4 + 1).
- Represent decompositions using a drawing or equation.
- Find the number that makes 10 when added to a given number, using numbers 1–9.
- Fluently add or subtract within 5.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Use objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations to represent addition and subtraction situations.
- Connect a word problem to a correct number sentence.

Use appropriate tools strategically.

- Use tools to visualize possible solutions.
- Use tools such as concrete models or diagrams to solve a math problem.
- Know when each of these tools might be helpful.
- Use technology tools to explore and deepen understanding of concepts.

Essential questions

- How many different number pairs can you make equal to10 (at least 5 pairs of numbers)?
- What are different ways you can represent the problem?
- How do you know if the number sentence represents/matches the problem?
- How can you record addition and subtraction problems using an equation or drawing with these given numbers (e.g., 4 and 6)?
- How can you add/subtract (to 5) quickly?

Written Curriculum

Counting and Cardinality K.CC

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

K.OA.5 Fluently add and subtract within 5.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

The range of students' mathematical abilities will vary depending on their exposure to the concepts. Students have very different experiences with number and counting. Some students have counted by rote while others may have had experience with place value. Some have been able to compose and decompose numbers.

Current Learning

Students represent addition and subtraction with objects, fingers, mental images, drawings, sounds, and acting out situations. In addition, they solve the operations using verbal explanations, expressions, and equations. Kindergarten students are encouraged to write equations, but it is not required. Students solve word problems within 10 by using the previously mentioned strategies. When given a number from 1 to 9, students find various addition equations, which equals 10. By the end of the year, students should be fluent in solving addition and subtraction equations to 5.

Future Learning

In grade 1, students will solve addition and subtraction word problems within 20. They will equate a symbol for the unknown number to represent the problem. They will add and subtract within 20 and fluently within 10. The meaning of the equal sign will be internalized and students will determine if the equations are true or false using the strategies of counting on and making 10. They determine the unknown whole number in addition and/or subtraction equations relating to three whole numbers. They will understand the properties of operations as strategies to add and subtract (e.g., If 8 + 3 = 11, then 3 + 8 = 11 also).

Additional Findings

According to *Curriculum Focal Points*, "Use concrete, pictorial, and verbal representations to develop an understanding of invented and conventional symbolic notations" (p. 24).

Principles and Standards for School Mathematics states, "An understanding of addition and subtraction can be generated when young students solve 'joining' and take-away problems by directly modeling the situation or by using counting strategies, such as counting on or counting back (Carpenter and Moser 1984)." (p. 83)

Kindergarten Mathematics, Quarter 4, Unit 4.4

Developing Concepts of Place Value—Composing/Decomposing Numbers into Tens and Some Ones (11–19)

Overview

Number of instructional days: 8 (1 day = 30 minutes)

Content to be learned

- Compose and decompose numbers from 11–19 into 10 ones and some more ones.
- Represent the composition or decomposition using an equation or drawing (e.g., 18 = 10 + 8).
- Develop understanding that two-digit numbers are composed of 10 ones and some more ones.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Explain how numbers 11–19 are related.
- Use drawings and objects to represent examples of quantities to 19.
- Use concrete objects or pictures to help conceptualize and make sense of the problem.

Attend to precision.

- Communicate understanding of quantities precisely to others.
- Use clear definitions when describing quantities to themselves and to others.

Essential questions

- How can you describe the number _____ (11–19)?
- How can you show the number _____ (11–19 as 10 ones and some more ones)?
- How can you represent this number _____ using tools?

Written Curriculum

Number and Operations in Base Ten	K.NBT

Work with numbers 11–19 to gain foundations for place value.

K.NBT.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

Students have very different experiences with number and counting. Some students have counted by rote while others may have had experience with place value. Some have been able to compose and decompose numbers. The range of students' mathematical abilities will vary depending on their exposure to the concepts.

Current Learning

Students compose and decompose numbers from 11–19 into 10 ones and some more ones. They are using objects or drawings to represent a number from 11–19. By the end of the year, students record their composition or decomposition with a drawing or an equation (i.e., 17 = 10 + 7). Students understand that these numbers are composed of 10 ones and 1, 2, 3, 4, 5, 6, 7, 8, or 9 ones. In kindergarten, "writing of equations is encouraged, but it is not required" (p. 9, CCSSM).

Future Learning

In first grade, students will understand that the two digits of a two-digit number represent the amounts of tens and ones. Students now call the bundle of 10 ones a "ten." They compose and decompose a number from 11–19 into a 10 and 1, 2, 3, 4, 5, 6, 7, 8, or 9 ones. They will refer to the numbers 10, 20, 30, 40, 50, 60, 70, 80, and 90 as 1, 2, 3, 4, 5, 6, 7, 8, or 9 tens (and 0 ones). Students will use place value understanding to add and subtract.

Additional Findings

In *Principles and Standards for School Mathematics*, students "develop a sense of whole numbers and represent and use them in flexible ways, including related, composing, and decomposing numbers." (p. 78)

Kindergarten Mathematics, Quarter 4, Unit 4.5

Comparing and Composing Shapes

Overview

Number of instructional days: 7 (1 day = 30 minutes)

Content to be learned

- Compare and analyze two- and three-dimensional shapes in different sizes and orientations.
- Model objects in the environment.
- Construct more complex shapes by composing simple shapes.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Flexibly use different properties of shapes to construct more complex shapes.
- Use concrete objects or pictures to help conceptualize and solve a problem.

Model with mathematics.

- Use basic shapes and spatial reasoning to model objects.
- Construct a larger shape using two or more simple shapes.
- Represent objects in the environment with shapes.
- Analyze relationships of two shapes to draw conclusions.

Essential questions

- How would you compare two shapes (e.g., a square and a rectangle)?
- How would you describe this shape? (e.g., cylinder)
- Using these materials, how can you create a _____? (e.g., triangle)
- How can you create a _____ (trapezoid) using _____ (triangles)?

Written Curriculum

Geometry	**K.G**

Analyze, compare, create, and compose shapes.

K.G.4 Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).

K.G.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.

K.G.6 Compose simple shapes to form larger shapes. For example, "Can you join these two triangles with full sides touching to make a rectangle?"

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

Children start forming concepts of shape long before they come to school. Students have very different experiences with geometry and spatial reasoning. They may have examined shapes of objects and discussed their relative positions. They have found shapes in their environment and described them in their own words. They may have built pictures and designs by combining two- and three-dimensional shapes. Many students have solved problems such as decided which piece fit into a space of a puzzle. The range of students' mathematical abilities will vary depending on their exposure to the concepts.

Current Learning

In kindergarten, students learn about two- and three-dimensional shapes. They learn to identify and describe shapes in their environment. They identify shapes based on their relative positions, such as above, below, beside, in front of, behind, and next to. Students correctly names shapes regardless of their orientation or overall size. They learn to differentiate a two-dimensional shape by lying in a "flat" plane and a three-dimensional shape as a "solid."

In this unit, further exploration of shapes requires students to analyze, compare, create, and compose shapes. Students use informal language to describe similarities, differences, attributes, and parts of shapes. They model shapes viewed in the world by drawing shapes and building shapes from components (e.g., sticks and clay balls). Students compose simple shapes to form more complex shapes.

Future Learning

In grade 1, students will distinguish defining attributes versus nondefining attributes. They will build and draw shapes to possess defining attributes. They will compose two- and three-dimensional shapes to create a composite shape and compose new shapes from the composite shape.

Additional Findings

In *Curriculum Focal Points*, "Children interpret the physical world with geometric ideas and describe it with corresponding vocabulary. They identify, name, and describe a variety of shapes, presented in a variety of ways, as well as such three-dimensional shapes as spheres, cubes, and cylinders. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes." (p. 12)

In *Principles and Standards for School Mathematics*, "Students should use their notions of geometric ideas to become more proficient in describing, representing, and navigating their environment. They should learn to represent two- and three-dimensional shapes through drawings, block constructions, dramatizations, and words. They should explore shapes by decomposing them and creating new ones." (p. 97)