Grade 3 Mathematics, Quarter 2, Unit 2.1

Measurement—Time and Mass

Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned

- Tell and write time to the nearest minute.
- Solve problems involving addition and subtraction of time intervals in minutes.
- Solve one-step word problems using addition and subtraction by measuring and estimating mass and volume of objects using standard units.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities and relationships in problem situations.
- Break down a word problem into components for better understanding.
- Translate a word problem into a number sentence in order to accurately solve.

Model with mathematics.

- Make connections to everyday life.
- Make connections to previously learned strategies.
- Apply mathematical understanding to solve problems.

Use appropriate tools strategically.

- Understand how and when to use tools.
- Indentify and use tools to visualize possible solutions.
- Use a number line to solve problems involving elapsed time.

Attend to precision.

- Use precise math terms, symbols, and units of measure.
- Calculate accurately and efficiently.
- Use clear and formulated explanations in communicating reasoning to others.

Essential questions

- What do you need to know to accurately tell time to the nearest minute?
- How can you use a number line to show elapsed time when solving a word problem?
- When solving a word problem, how can you measure and estimate the mass/volume of objects?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data	**3.MD**

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

3.MD.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

3.MD.2 Measure and estimate ~~liquid volumes and~~ masses of objects using standard units of grams (g), kilograms (kg), ~~and liters (l)~~.[6] Add, subtract~~, multiply, or divide~~ to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[7]

[6] Excludes compound units such as cm^3 and finding the geometric volume of a container.

[7] Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2).

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.

Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 2. students learned to tell time to the nearest five minutes. They were introduced to pictographs and bar graphs. Students learned attributes of various polygons.

Current Learning

In grade 3, students tell time to the nearest minute in problem-solving tasks. They measure and estimate the mass of objects solving word problems using addition and subtraction.

Later in the year, students measure and estimate liquid volumes, solving word problems using multiplication and division.

Future Learning

In grade 4, students will measure and convert length using kilometers, meters, and centimeters; mass using pounds and ounces; volume using liters; and time using hours and seconds. They will use the four operations to solve word problems involving distance, time, volume, and mass.

Students will apply area and perimeter formulas for rectangles. They will make line plots to display data of measurement to solve problems.

Additional Findings

According to *Curriculum Focal Points*, students form an understanding of perimeter as a measureable attribute and select appropriate units, strategies, and tools to solve problems involving perimeter.

Grade 3 Mathematics, Quarter 2, Unit 2.2

Equivalent Fractions

Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned

- Recognize fractions as numbers.
- Understand and determine if two fractions are equivalent using visual models.
- Express whole numbers as fractions.
- Recognize fractions that are equal to whole numbers.
- Generate and recognize simple equivalent fractions.
- Compare two fractions with same numerator or denominator by reasoning about their size.
- Use symbols <, >, and = and visual models to represent comparisons of fractions.
- List equivalent fractions and explain, using visual models.
- Understand that when comparing fractions the whole must be the same.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Justify answers and explain thinking.
- Look at other students' solutions for additional strategies.

Look for and express regularity in repeated reasoning.

- Use prior knowledge to find a solution.
- Look for patterns and repetitions when solving problems.
- Evaluate the reasonableness of the projected answer.
- Become efficient with comparing fractions.

Essential questions

- What is a fraction?
- How do you know two fractions are equivalent?
- How can you use visual models to show equivalent fractions?
- How can you express a whole number as a fraction?
- How can fractions with the same numerator or the same denominator compare in size?
- How can you demonstrate the value of fractions using <, >, or =?
- How do you know when a fraction is equivalent to a whole number?
- What do you need to know when comparing two fractions with the same numerator/ denominator?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions[5]	3.NF

[5] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

Develop understanding of fractions as numbers.

3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope

3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In grade 2, students created line plots where scale was marked off in whole number units. They partitioned circles and rectangles into two, three, or four equal shares using the words *halves, thirds, half of, a third of,* etc. Students described the whole as a two halves, three thirds, and four fourths. They used <, >, and = to compare whole numbers.

Current Learning

In grade 3, students recognize and understand that fractions are numbers. They learn and understand fractions on a number line. Students develop an understanding of fractional equivalence. They compare two fractions with the same numerator or denominator using symbols <, >, and = and visual models. Students generate simple equivalent fractions (e.g., 1/2 = 2/4; 4/6 = 2/3). Students explain why the fractions are equivalent using a visual model. They recognize that fractions are equivalent to whole numbers and express whole numbers as fractions.

Future Learning

In grade 4, students will explain why a fraction *a/b* is equivalent by using visual fraction models, with attention to how the number and size parts differ, even though two fractions are the same size. They will compare two fractions with different numerators and different denominators using <, >, and =. Students will use common denominators utilizing visual models, and they will employ benchmark fractions to determine equivalence.

Additional Findings

According to *Curriculum Focal Points*, "The students understand and use models including the number line to identify equivalent fractions."(p. 15) In addition, "Students use models, benchmarks, and equivalent forms to judge the size of fraction." (p. 29)

According to the *Principles and Standards for School Mathematics*, "Students should develop strategies for ordering and comparing fractions." (p. 150)

Grade 3 Mathematics, Quarter 2, Unit 2.3

Using Commutative and Distributive Properties

Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned

- Apply properties of operations as strategies to multiply.
- Multiply within 100.
- Multiply one-digit whole numbers by multiples of 10 in the range 10–90, understanding that as the one-digit number increases, the total product increases by 10, relatively.
- Know from memory products of two 1-digit numbers.
- Fluently add and subtract within 1,000 using strategies and algorithms based on place value and properties of operations.

Mathematical practices to be integrated

Attend to precision.

- Use math terms and symbols precisely.
- Calculate accurately and efficiently.
- Formulate clear explanations and communicate reasoning.

Look for and make use of structure.

- Decompose numbers to solve problems.
- Make connections between addition and multiplication.
- Gain a sense of understanding of properties of numbers (commutative and distributive).
- Look at problem and use what is known to find simpler solutions.

Look for and express regularity in repeated reasoning.

- Use prior knowledge to find a solution.
- Look for patterns and repetitions when solving problems.
- Draw conclusions and find a path to solve problems.
- Evaluate the reasonableness of the projected answer.

Essential questions

- How can you show the strategies you use to multiply numbers?
- How can you demonstrate multiplication equations within 100?
- How can you show two ways to explain multiplication of a one-digit by a multiple of 10?
- How can you accurately add and subtract within 1,000 using at least two strategies?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	3.OA

Understand properties of multiplication and the relationship between multiplication and division.

3.OA.5 Apply properties of operations as strategies to multiply ~~and divide~~.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.)* ~~*3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.)*~~ *Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*

[2] Students need not use formal terms for these properties.

Multiply and divide within 100.

3.OA.7 ~~Fluently~~ multiply ~~and divide~~ within 100, using strategies such as ~~the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or~~ properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Number and Operations in Base Ten	3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.[4]

[4] A range of algorithms may be used.

3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Common Core Standards for Mathematical Practice

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8

equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In grade 2, students progressed from using addition to repeated addition (foundational to multiplication) by finding the total number of objects in arrays up to 5×5. They counted objects by twos using skip counting and repeated addition.

Current Learning

In Unit 1.4, students interpret products as whole numbers. They use multiplication to solve word problems involving equal groups, arrays, and drawings with equations of unknown symbols. Students focus on using the Commutative and Distributive Properties of Multiplication to develop computational strategies.

Students apply properties of operations as strategies to multiply, multiply within 100, and multiply one-digit whole numbers by multiples of 10 in the range 10–90. They know from memory all products of two 1-digit numbers by the end of the school year.

Students fluently add and subtract within 1,000 using strategies and algorithms based on place value and properties of operations.

Future Learning

In grade 4, students will interpret multiplication equations as a comparison, multiply a whole number up to four digits by a one-digit whole number, and multiply two 2-digit numbers using strategies based on place value and the properties of operations.

Students will multiply and divide to solve word problems with an unknown symbol and will be able to distinguish multiplicative comparisons from additive comparisons. They will use multistep word problems using the four operations.

Students will fluently add and subtract multidigit whole numbers using the standard algorithm.

Additional Findings

According to *Curriculum Focal Points*, "They use properties of addition and multiplication to multiply whole numbers and apply increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving basic facts." (p. 15)

According to *PARCC: K–5, Number and Operations in Base Ten*, "The special role of 10 in the base-ten system is important in understanding multiplication of one-digit numbers with multiples of 10." (p. 11)

According to *Principles and Standards for School Mathematics*, third graders "develop fluency in adding subtracting, multiplying, and dividing whole numbers." (p. 392)

Grade 3 Mathematics, Quarter 2, Unit 2.4

Relationships Between Multiplication and Division

Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned

- Solve division equations with whole numbers having no remainders.
- Solve multiplication and division equations with an unknown number (fact families).
- Understand division as an unknown-factor problem.
- Understand the associative property of multiplication.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Read and understand word problems.
- Plan a strategy to solve problems.
- Assess solutions for reasonableness and change strategies if necessary.

Construct viable arguments and critique the reasoning of others.

- Justify answers and explain thinking.
- Look at other students' solutions for additional strategies.

Look for and make use of structure.

- Relate unknown facts to simpler known facts.
- Look for patterns and repetitions when solving problems.
- Make connections to previously learned strategies.

Essential questions

- How do you solve a division problem using whole numbers without remainders?
- When given two numbers, how do you show a fact family using multiplication and division?
- How does a fact family help you solve a division problem?
- How can you solve a multiplication problem with an unknown number?
- How can you solve a division problem with an unknown factor?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	**3.OA**

Represent and solve problems involving multiplication and division.

3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*

3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = □ ÷ 3, 6 × 6 = ?.*

Understand properties of multiplication and the relationship between multiplication and division.

3.OA.5 Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*

[2] Students need not use formal terms for these properties.

3.OA.6 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In grade 2, students progressed from using addition, to multiplication by finding the total number of objects in arrays up to 5×5. They counted objects by twos using skip counting and used repeated addition to find the total.

Current Learning

In grade 3, students use division of whole numbers without remainders. They determine an unknown whole number in a multiplication or division sentence. Students understand division as an unknown-factor problem. Students develop an understanding of the associative property of multiplication.

Future Learning

In grade 4, students will interpret multiplication equations as a comparison, will multiply a whole number up to four digits by a one-digit whole number, and will multiply two 2-digit numbers using strategies based on place value and the properties of operations.

Students will multiply and divide to solve word problems with an unknown symbol, and will be able to distinguish multiplicative comparisons from additive comparisons. Students will use multistep word problems using the four operations.

Additional Findings

According to *Principles and Standards for School Mathematics*, third graders "… develop fluency in adding subtracting, multiplying, and dividing whole numbers." (p. 392)