Grade 3 Mathematics, Quarter 3, Unit 3.1

Multiplication and Division Problem Solving

Overview

Number of instructional days: 15 (1 day = 45 minutes)

Content to be learned

- Solve word problems involving multiplication and division situations within 100.
- Use strategies to multiply and divide within 100.
- Know from memory products of two 1-digit numbers.
- Solve two-step word problems using the four operations.
- Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
- Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.
- Represent problems using equations with a symbol/letter for the unknown.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Read and understand word problems.
- Plan a strategy to solve problems.
- Assess solutions for reasonableness and change strategies if necessary.

Model with mathematics.

- Use symbols to represent the unknown quantity.
- Use drawings and equations to represent the problem.
- Accurately explain the solution representation.

Look for and make use of structure.

- Compose and decompose numbers to simplify solutions.
- Identify patterns in the multiplication facts.
- Explain patterns in the properties of numbers (commutative and distributive).

Look for and express regularity in repeated reasoning.

- Use known facts to find a solution.
- Look for and use patterns and repetitions when solving problems.
- Evaluate the reasonableness of projected answer.

Essential questions

- What strategy would you use to solve a multiplication word problem?
- What strategy would you use to solve a division word problem?
- How can you solve a two-step word problem with at least two operations?
- How do you use mental computation and estimation strategies to check your answer?
- What patterns do you notice in the multiplication/division facts?
- How can you use a known multiplication fact to solve an unknown fact?
- How can you use an equation to represent the problem with an unknown quantity?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	3.OA

Represent and solve problems involving multiplication and division.

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, ~~and measurement quantities,~~ e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

[1] See Glossary, Table 2.

Multiply and divide within 100.

3.OA.7 ~~Fluently~~ multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3]

[3] This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In grade 2, students progress from using addition, to multiplication by finding the total number of objects in arrays up to 5×5. They counted objects by twos, fives, and tens, using skip counting and repeated addition. Students estimated lengths of units of standard measurement.

Current Learning

Students use multiplication and division within 100 to solve word problems in situations involving equal groups and arrays. They multiply and divide within 100, using strategies such as the relationship between multiplication and division. By the end of grade 3, students know from memory all products of two one-digit numbers.

Students solve two-step word problems using the four operations and represent these problems using equations with a letter standing for the unknown quantity. They assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Students identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.

Future Learning

In grade 4, students will interpret multiplication equations as a comparison. They will multiply a whole number up to four digits by a one-digit whole number, and will multiply two 2-digit numbers using strategies based on place value and the properties of operations.

Students will multiply and divide to solve word problems with an unknown symbol. They will be able to distinguish multiplicative comparisons from additive comparisons. Students will use multistep word problems using the four operations. Students will generate a number pattern that follows a given rule.

Additional Findings

According to *Curriculum Focal Points*, "They use properties of addition and multiplication to multiply whole numbers and apply increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving basic facts." (p. 15)

According to *Principles and Standards for School Mathematics*, third graders "… develop fluency in adding subtracting, multiplying, and dividing whole numbers." (p. 392)

Grade 3 Mathematics, Quarter 3, Unit 3.2

Measurement—Plane Figures: Area and Attributes

Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned

- Understand that a square with a side length of one unit is called "a unit square" and is said to have "one square unit" of area.
- Recognize area as an attribute of plane figures, consisting of square units.
- Understand that shapes in different categories may share attributes of subcategories.

Mathematical practices to be integrated

Use appropriate tools strategically.

- Identify tools that are relevant and helpful.
- Use tools to deepen understanding of area.
- Use tools such as color tiles to create or solve problems.
- Use tools to visualize possible solutions and/or to detect errors.

Attend to precision.

- Use math terms and symbols precisely.
- Calculate accurately and efficiently.
- Measure areas without gaps or overlaps.
- Formulate clear explanations and communicate reasoning.

Essential questions

- What is area measurement?
- How can you use visual models to demonstrate square units in a plane figure?
- Why is area measured in square units?
- How can you compare categories with shared attributes?
- How do you find the area of a given object?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data	3.MD

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

Geometry	3.G

Reason with shapes and their attributes.

3.G.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Common Core Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem.

They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 2, students used standard units of measure. They measured the length of an object using rulers, yardsticks, meter sticks, and measuring tapes. Students measured the length of an object twice, using length units of different lengths for the two measurements. They recognized and drew shapes having specific attributes. Students identified triangles, quadrilaterals, pentagons, hexagons, and cubes.

Current Learning

In Unit 1.4, students relate the area of a rectangle to multiplying side lengths. They recognize area as an attribute of plane figures and understand concepts of area measurement. Students understand that shapes in different categories may share attributes. They recognize rhombuses, rectangles, and squares as examples of quadrilaterals.

Future Learning

In grade 4, students will apply area and perimeter formulas for rectangles in real-world and mathematical problems. They will draw points, lines, line segments, rays, angles, and perpendicular and parallel lines. Students will classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines. They will recognize right triangles and lines of symmetry.

Additional Findings

According to *Curriculum Focal Points*, "Students describe, analyze, compare, and classify two-dimensional shapes by their sides and angles and connect these attributes to definitions of shapes." (p. 15)

According to *Principles and Standards for School Mathematics,* students "understand such attributes as length, area, weight, volume, and size of angle and select the appropriate type of unit for measuring each attribute; understanding the need for measuring with standard units and become familiar with standard units in the customary and metric systems." (p. 398)

Grade 3 Mathematics, Quarter 3, Unit 3.3

Measurement—Area, Including Problem Solving

Overview

Number of instructional days: 15 (1 day = 45 minutes)

Content to be learned

- Measure area by counting unit squares.
- Multiply side lengths to find areas of rectangles using real-world and mathematical problems.
- Solve problems in area using repeated addition.
- Find the area of a shape made up of two or more right-angle polygons (rectilinear polygon) by adding the individual areas of each polygon.
- Solve word problems using multiplication involving equal groups, arrays, and measurement quantities, including using a symbol for an unknown number.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities and relationships in problem situations.
- Break down a word problem into components for better understanding.
- Translate a word problem into a number sentence to accurately solve.
- Translate a number sentence into a word problem.

Model with mathematics.

- Make connections to everyday life.
- Make connections to previously learned strategies.
- Apply mathematical understanding to solve problems.

Attend to precision.

- Use math terms and symbols precisely.
- Calculate accurately and efficiently.
- Formulate clear explanations and communicate reasoning.

Essential questions

- How do you find the area of a rectangle using square units?
- How can you find the area of a shape made up of multiple polygons having right angles?
- How can you use multiplication or addition to solve problems involving area?
- How can you solve a multiplication problem with a symbol representing an unknown number?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data	3.MD

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

3.MD.7 Relate area to the operations of multiplication and addition.

b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Operations and Algebraic Thinking	3.OA

Represent and solve problems involving multiplication and division.

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

[1] See Glossary, Table 2.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to

solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 2, students partitioned a rectangle into rows and columns of same-sized squares and counted to find the total number of them. They used addition to find the total number of objects arranged in rectangular arrays with up to five rows and up to five columns. Students solved addition and subtraction problems using drawings and equations with unknowns in all positions.

Current Learning

In grade 3, students measure area by counting unit squares. They multiply side lengths to find areas of rectangles using real-world and mathematical problems. Students solve problems in area using repeated addition. They find the area of a shape made up of two or more right-angle polygons (rectilinear polygon) by adding the individual areas of each polygon. Students solve word problems using multiplication involving equal groups, arrays, and measurement quantities with unknowns in all positions.

Future Learning

In grade 4, students will apply area and perimeter formulas for rectangles in real-world and mathematical problems. They will solve multistep word problems with four operations using unknowns in all positions.

Additional Findings

According to *Principles and Standards for School Mathematics*, students “develop strategies for estimating the perimeters, areas, and volumes of irregular shapes.” (p. 398)