

Grade 3 Mathematics, Quarter 4, Unit 4.1

Measurement—Liquid Volume

Overview

Number of instructional days: 7 (1 day = 45 minutes)

Content to be learned

- Measure and estimate liquid volumes and mass of objects.
- Solve one-step word problems using addition, subtraction, multiplication, and division involving mass and volume.

Mathematical practices to be integrated

Use appropriate tools strategically.

- Identify tools that are relevant and helpful.
- Use tools to deepen understanding of math concepts.
- Use tools to create or solve problems.
- Use tools to visualize possible solutions and/or to detect error.

Attend to precision.

- Use math terms and symbols precisely.
- Calculate accurately and efficiently.
- Formulate clear explanations and communicate reasoning.

Essential questions

- What ways could you represent your solution to word problems with masses or volumes with the same unit of measure?
- How can you measure liquid volume or masses of objects using standard units?
- How can you estimate liquid volume or masses of objects using standard units?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data

3.MD

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

- 3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).⁶ Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.⁷

⁶ Excludes compound units such as cm³ and finding the geometric volume of a container.

⁷ Excludes multiplicative comparison problems (problems involving notions of “times as much”; see Glossary, Table 2).

Common Core Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 2, students measured and estimated the length of objects using standard U.S. customary units and metric units, and appropriate tools, such as ruler, yardstick, meter stick, and measuring tape. They solved word problems using length.

Current Learning

In grade 3, students measure and estimate liquid volumes and masses of objects using standard units including grams, kilograms, and liters. Students add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units (e.g., by using drawings, such as a beaker with a measurement scale, to represent the problem).

Future Learning

In grade 4, students will use four operations to solve word problems involving liquid volumes and masses of objects using standard measurements. Students will know the relative sizes of measurement units within one system of units (e.g., U.S. customary unit or metric unit).

Additional Findings

According to *Principles and Standards for School Mathematics*, “Students should have begun to develop an understanding of what it means to measure an object, identifying an attribute to be measured ... they should have had many experiences measuring length and explored ways to measure liquid volume, weight and time.” (p. 171)

Grade 3 Mathematics, Quarter 4, Unit 4.2

Representing and Interpreting Data Using Picture Graphs and Bar Graphs

Overview

Number of instructional days: 7 (1 day = 45 minutes)

Content to be learned

- Represent a data set with several categories by drawing a scaled picture graph.
- Draw a scaled bar graph to represent a data set with several categories.
- Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs.

Mathematical practices to be integrated

Model with mathematics.

- Make connections to everyday life when solving problems.
- Make connections to previously learned strategies.

Use appropriate tools strategically.

- Identify tools that are relevant and helpful.
- Use technology tools to deepen understanding of math concepts.
- Use tools to create or solve problems.
- Use tools to visualize possible solutions so errors can be detected.

Attend to precision.

- Demonstrate precise use of math terms, symbols, and units of measure.
- Calculate accurately and efficiently.
- Formulate clear explanations and communicate reasoning.

Essential questions

- How can you draw a scaled picture to represent data?
- How can you draw a bar graph to represent data?
- How do you solve one- and two-step problems using information from graphs?
- How do you describe the data between two categories of “how many more” and “how many less”? (e.g., how many more students like math more than science?)
- How did you decide the scale to use for your graph?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data

3.MD

Represent and interpret data.

- 3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards*Prior Learning*

In grade 2, students made picture and bar graphs with a single unit scale to represent data up to four categories. They solved simple put together, take apart, and compare problems using information in the bar graph. Students made line plots to show measurement in whole number units.

Current Learning

In grade 3, students draw a scaled picture and bar graph to represent a data set with several categories. They solve one- and two-step “how many more?” and “how many less?” problems.

Future Learning

In grade 4, students will make a line plot to display a set of measurements in fractions of a unit. Students will use information from line plots to solve problems involving addition and subtraction of fractions.

Additional Findings

According to *Principles and Standards for School Mathematics*, “Students should become familiar with a variety of representations such as table, line plots, bar graphs, and line graphs by creating them, watching their teacher create them, and observing representations found in their environment.” (p. 178)

Grade 3 Mathematics, Quarter 4, Unit 4.3

Geometric Measurement— Perimeter of Polygons

Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned

- Solve real-world and mathematical problems involving perimeters of polygons.
- Find the perimeter given the side lengths.
- Solve problems involving perimeter, finding an unknown side length.
- Recognize rectangles with the same perimeter and different areas or with the same area and different perimeters.

Mathematical practices to be integrated

Model with mathematics.

- Make connections to everyday life when solving problems.
- Make connections to previously learned strategies.

Use appropriate tools strategically.

- Identify tools that are relevant and helpful.
- Use technology tools to deepen understanding of math concepts.
- Use tools to create or solve problems.
- Use tools to visualize possible solutions so errors can be detected.

Attend to precision.

- Demonstrate precise use of math terms, symbols, and units of measure.
- Calculate accurately and efficiently.
- Formulate clear explanations and communicate reasoning.

Essential questions

- How do you solve real-world problems involving perimeter of polygons?
- How do you calculate the perimeter of an object?
- How do you explain the area of an object?
- If an object has an unknown side length, how can you find that length?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data

3.MD

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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6 **Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 2, students measured length using standard U.S. customary and metric units. They solved word problems and used addition and subtraction involving lengths that are given.

Current Learning

In grade 3, students solve real-world and mathematical problems involving perimeter of polygons, including finding the perimeter given the side lengths, and finding an unknown side length. Students show rectangles with same perimeter and different perimeters and areas.

This is a good place to reinforce addition, subtraction, and measurement skills using real-world problems within the perimeter context.

Future Learning

In grade 4, students will apply the area and perimeter formulas for rectangles in real-world and mathematical problems.

Additional Findings

According to *Principles and Standard for School Mathematics*, “Students in grades 3–5 should measure objects and space in their classroom or use maps to determine locations and distances around their community.” (p. 171)

Grade 3 Mathematics, Quarter 4, Unit 4.4

Computation—Fluency in Addition and Subtraction to 1,000

Overview

Number of instructional days: 8 (1 day = 45 minutes)

Content to be learned

- Use strategies and algorithms based on place value to fluently add and subtract within 1,000.
- Fluently add and subtract within 1,000 using strategies and algorithms based on properties of operations.
- Use strategies and algorithms based on the relationship between addition and subtraction to fluently add and subtract within 1,000.

Mathematical practices to be integrated

Attend to precision.

- Use of math terms and symbols precisely.
- Calculate accurately and efficiently.
- Formulate clear explanations and communicate reasoning.

Look for and make use of structure.

- Develop greater number sense; able to decompose numbers.
- Identify patterns in addition and subtraction problems.
- Gain a sense of understanding of properties of numbers (commutative and distributive).
- Look at problem and use what is known to find simpler solutions.

Look for and express regularity in repeated reasoning.

- Use prior knowledge to find a solution.
- Look for patterns and repetitions when solving problems.
- Draw conclusions and find a path to solve problems.
- Evaluate the reasonableness of projected answer.

Essential questions

- How can you use place value to add/subtract a three-digit number?
- How can your understanding of addition be used when you are subtracting?
- How can you accurately add and subtract within 1,000 using at least two strategies?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten

3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.⁴

⁴ A range of algorithms may be used.

- 3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Common Core Standards for Mathematical Practice

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In grade 2, students fluently added and subtracted within 100 using place value, and properties of operations. They added and subtracted within 1,000 using concrete models or drawing and strategies based on place value and properties of operations.

Current Learning

Third graders use prior learning to develop fluency in addition and subtraction within 1,000. They use their understanding of place value and properties of operations to add and subtract.

Future Learning

In grade 4, students will fluently add and subtract multidigit whole numbers using standard algorithm. They will use all four operations with whole numbers to solve multistep problems.

Additional Findings

According to *Curriculum Focal Points*, “They use properties of addition and multiplication to multiply whole numbers and apply increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving basic facts.” (p. 15)

According to *Principles and Standards for School Mathematics*, third graders “develop fluency in adding, subtracting, multiplying, and dividing whole numbers.” (p. 392)

Grade 3 Mathematics, Quarter 4, Unit 4.5

Multiplication and Division Fluency Within 100

Overview

Number of instructional days: 8 (1 day = 45 minutes)

Content to be learned

- Multiply and divide fluently within 100 using properties of operations as strategies.
- Know from memory all products of two one-digit numbers.
- Use the relationship of multiplication and division as a strategy to fluently multiply and divide within 100.

Mathematical practices to be integrated

Attend to precision.

- Explain reasoning clearly.
- Calculate accurately and efficiently.
- Formulate clear explanations and communicate reasoning.

Look for and make use of structure.

- Decompose numbers to known facts to find unknown facts.
- Recognize and identify patterns in multiplication facts.
- Make connections beyond addition to multiplication.
- Understand and apply properties of numbers (commutative and distributive).
- Look at problem and use what is known to find simpler solutions.

Look for and express regularity in repeated reasoning.

- Use known facts to find a solution.
- Look for patterns and repetitions in multiplication and division when solving problems.
- Evaluate the reasonableness of projected answer.

Essential questions

- How can you show the strategies you use to multiply numbers?
- How can you demonstrate multiplication equations within 100?
- How can you show two ways to explain multiplication of a one-digit by a multiple of 10?
- How can you show the strategies you use to divide numbers?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking

3.OA

Multiply and divide within 100.

- 3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Common Core Standards for Mathematical Practice

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In grade 2, students used repeated addition to find total number of objects arranged in rectangular arrays with up to five rows and up to five columns.

Current Learning

In grade 3, students fluently multiply and divide within 100 using strategies such as the relationship between multiplication and division or properties of operations. Students know from memory all products of two one-digit numbers. Students apply the properties of operations to multiply and divide. Students apply their understanding of multiplying one-digit whole numbers by multiples of 10 from unit 2.3, 3.NBT.3, when multiplying within 100.

Future Learning

In grade 4, students will multiply and divide to solve word problems involving multiplicative comparison. Students will interpret a multiplication equation as a comparison. Students will solve multistep word problems using four operations.

Additional Findings

According to *Curriculum Focal Points*, “They use properties of addition and multiplication to multiply whole numbers and apply increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving basic facts.” (p. 15)

According to *PARCC: K–5, Number and Operations in Base Ten*, “The special role of 10 in the base-ten system is important in understanding multiplication of one-digit numbers with multiples of 10.” (p. 11)

According to *Principles and Standards for School Mathematics*, third graders “develop fluency in adding subtracting, multiplying, and dividing whole numbers.” (p. 392)

