Grade 4 Mathematics, Quarter 2, Unit 2.1

Operations and Algebraic Thinking in Problem Solving

Overview

Number of instructional days: 11 (1 day = 60 minutes)

Content to be learned

- Find whole number quotients with up to four-digit dividends and one-digit divisors.
- Use strategies based on place value, properties of operations, and the relation between multiplication and division.
- Illustrate and explain calculations by using equations, rectangular arrays, and/or area models.
- Multiply or divide to solve word problems involving multiplicative comparison.
- Use drawings and equations with a symbol for the unknown number to represent the problem.
- Solve multistep word problems with whole numbers using the four operations.
- Assess the reasonableness of answers using mental math, estimation, and rounding.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Justify conclusions and communicate them to others.
- Determine how their argument applies and make justifications.

Make sense of problems and persevere in solving them.

- Explain the meaning of a problem and look for entry points to its solution.
- Monitor and evaluate their progress and change course if necessary.
- Check answers and ask if they make sense.

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships.
- Construct viable arguments and critique the reasoning of others.

Essential questions

- How can you divide numbers using place value?
- How can you illustrate division of whole numbers using graphic representations?
- How do you solve a multistep word problem with whole numbers using equations with an unknown?
- Given a multiplication or division word problem, how can multiplicative comparison be used to solve?
- What methods can you use to assess the reasonableness of your answers?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten[2] 4.NBT

[2] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.6 Find whole-number quotients ~~and remainders~~ with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Operations and Algebraic Thinking 4.OA

Use the four operations with whole numbers to solve problems.

4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1]

[1] See Glossary, Table 2.

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations~~, including problems in which remainders must be interpreted~~. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Clarifying the Standards

Prior Learning

In grade 3, students used multiplication and division within 100 to solve word problems involving equal groups, arrays, and measurement quantities. They solved two-step word problems using the four operations.

Current Learning

In grade 4, students apply place value understanding to multidigit whole numbers. They use place value understanding and properties of operations to perform multidigit whole number multiplication and division equations and word problems. In grade 4, an unknown quantity is referenced with a symbol or letter, but is not yet referred to as a variable. The standard algorithm for division is not addressed.

Future Learning

In grade 5, students will find whole number quotients of whole numbers with up to four-digit dividends and two-digit divisors using the strategies taught in grade 4.

Additional Findings

According to *Principles and Standards for School Mathematics,* for students in grades 3–5,

> "Multiplicative reasoning should become a focus. Multiplicative is more than just doing multiplication and division. It is about understanding situations in which multiplication or division is the appropriate operation." (p. 143)
>
> "Development of number sense should continue, with a focus on multiplication and division. Their understanding of the meaning of these operations should grow deeper as they encounter a range of representations and problem situations, learn about the properties of this operation and develop fluency in this whole number computation." (p. 149)
>
> "Further meaning for multiplication should develop as students build on and describe area models. The area model is important because it helps students develop an understanding of the multiplicative properties." (p. 152)
>
> "At this grade band, the idea and usefulness of a variable (represented by of a box, a letter, or a symbol) should also be emerging and developing more fully." (p. 161)

Grade 4 Mathematics, Quarter 2, Unit 2.2

Division with Remainders

Overview

Number of instructional days: 10 (1 day = 60 minutes)

Content to be learned

- Find whole number quotients and remainders with up to four-digit dividends and one-digit divisors.
- Use strategies based on place value, the properties of operations, and the relationship between multiplication and division.
- Illustrate and explain the calculations by using equations, rectangular arrays, and/or area models.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Explain the meaning of a problem to themselves and look for entry points to its solution.
- Monitor and evaluate their progress and change course if necessary.

Reason abstractly and quantitatively.

- Make sense of quantities and their relationship in problem situations.
- Create a coherent representation of the problem at hand considering the units involved and the meaning of quantities.

Essential questions

- What strategies can be used to find whole number quotients and remainders with up to four-digit dividends and one-digit divisors?
- Using equations, rectangular arrays, and/or area models, can you illustrate and explain division problems?
- How can you solve multistep word problems with whole numbers using the four operations?
- What does the remainder in a quotient represent?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten[2]	4.NBT

[2] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Operations and Algebraic Thinking	4.OA

Use the four operations with whole numbers to solve problems.

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into

the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Clarifying the Standards

Prior Learning

In grade 3, students developed an understanding of division of whole numbers within 100 through activities and problems involving equal-sized group arrays and area models and in finding an unknown factor in these situations.

Current Learning

In grade 4, students apply their understanding of models for division, place value, properties of operations (see Table 3 of CCSS) , and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multidigit dividends.

Future Learning

In grade 5, students will develop an understanding of why division procedures work based on the meaning of base-10 numerals and properties of operations. They will finalize fluency with multidigit division.

Additional Findings

According to *Principles and Standards for School Mathematics* for students in grades 3–5,

> "Multiplicative reasoning should become a focus. Multiplicative is more than just doing multiplication or division. It is about understanding situations in which multiplication or division is the appropriate operation." (p. 143)
>
> "Development of number sense should continue, with a focus on multiplication and division. Their understanding of the meaning of these operations should grow deeper as they encounter a range of representations and problem situations, learn about the properties of this operation and develop fluency in this whole number computation." (p. 149)
>
> "Where the results of division include a remainder … They should learn the meaning of a remainder by modeling division problems and exploring the size of remainders given a particular divisor." (p. 151)

Grade 4 Mathematics, Quarter 2, Unit 2.3

Area, Perimeter, and Measurement

Overview

Number of instructional days: 10 (1 day = 60 minutes)

Content to be learned

- Apply area formulas to find unknown lengths *or* areas for rectangles in real-world and mathematical problems.
- Apply perimeter formulas to find unknown lengths *or* perimeters for rectangles in real-world and mathematical problems.
- Know relative sizes of measurement units within the same system of measurement.
- Express measurements of a larger unit in terms of a smaller unit within the same system and state as a multiplicative comparison.
- Record measurement conversions in a two-column table.
- Use addition, subtraction, and multiplication to solve word problems involving distances, time, liquid volume, mass, and money that may require conversion of a larger unit into a smaller unit of measure.
- Represent measurement quantities when solving problems using diagrams such as a scaled number line.

Mathematical practices to be integrated

Use appropriate tools strategically.

- Consider available tools when solving a mathematical problem.
- Make sound decisions about when tools might be helpful.
- Detect possible errors by strategically using estimation and other mathematical knowledge.

Attend to precision.

- Communicate precisely to others using clear definitions.
- State the meaning of symbols and specify units of measure.
- Calculate accurately and efficiently.

Essential questions

- In what way could you represent this problem using a formula?
- By what means did you determine the measurement of the unknown quantity using your formula?
- What question is this formula/equation asking? (e.g., $20 = 5 \times \boldsymbol{l}$ is asking for the length of the rectangle that has an area of 20 square units)
- How could you represent the measurements using a drawing or diagram, such as a scaled number line, to show your measurements are related?
- What information is this problem asking you to find? What unit of measure will you need to use? How will you label your answer?
- What are you actually measuring? What units would you use to measure liquid volume?

- What is approximately 1 centimeter in length? 1 meter? 1 kilometer? 1 inch? 1 foot? 1 yard?
- How can you express a larger measurement unit in terms of a smaller unit of measurement? How can you state the relationship as a multiplicative comparison?
- How can you record equivalent measurements between two different units using a table? What patterns do you notice in your table?
- What are you actually measuring? What units would you use to measure mass?
- Why is it important to be consistent with the unit you are using when measuring?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data	**4.MD**

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*

4.MD.2 Use ~~the four operations~~ to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, ~~including problems involving simple fractions or decimals~~, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Common Core Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 3, students learned how to recognize area as an attribute of two-dimensional regions. They measured the area of a shape by finding the total number of same-sized units of area required to cover a shape. Students decomposed rectangles into arrays of squares to connect area to multiplication and to justify using multiplication to determine the area of a rectangle. They solved problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. Students recognized perimeter as an attribute of plane figures and distinguished between linear and area measures.

Current Learning

In grade 4, students solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit within the same system. They know the relative sizes of measurements within one system, including kilometers, meters, centimeters; kilograms, grams; pounds, ounces; liters, milliliters; hours, minutes, seconds. Students record equivalent measurements in a two-column table where multiplicative patterns can be seen and multiplicative comparison statements can be made. They apply the area and perimeter formulas for rectangles to real-word and mathematical problems in both a doing and undoing fashion (e.g., when given the lengths, find the area; when given the area and one length, find the missing length). By the end of the year, students use the four operations to solve word

problems involving distances, intervals of time, liquid volumes, masses of objects, and money that may require measurement conversions.

Future Learning

In grade 5, students will recognize volume as an attribute of three-dimensional space, and they select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. Students will decompose three-dimensional shapes and find volume. They will apply the formulas for volume of rectangular prisms. Students will convert among different-sized measurement units within a given measurement system, extending to expressing a smaller unit in terms of a larger unit, which involves division. Lastly, students will use these conversions when solving multistep, real-world problems.

Additional Findings

According to *Principles and Standards for School Mathematics*, "In learning about measurement and how to measure, students should be actively involved, drawing on familiar and accessible context." (p. 171)

According to *A Research Companion to Principles and Standards for School Mathematics*,

- "Other aspects of area measurement remain problematic, even though students can recall standard formulas for finding the areas of squares and rectangles." (p. 185)
- "Rectangular area is treated in schooling as a simple matter of multiplying lengths, but the research suggest that many students in the elementary grades do not 'see' this product as a measurement." (p. 185)
- "Measurement is essential for developing an understanding of the natural world (Crosby, 1977). By quantifying and otherwise mathematizing nature (Kline, 1980), students can model the natural world even at an earlier age." (p. 188)

Grade 4 Mathematics, Quarter 2, Unit 2.4

Developing an Understanding of Fractions

Overview

Number of instructional days: 10 (1 day = 60 minutes)

Content to be learned

- Understand why a fraction is equivalent.
- Explain why a fraction is equivalent to another fraction using visual fraction models.
- Recognize and generate equivalent fractions.
- Compare two fractions with different numerators and different denominators by creating common denominators or numerators.
- Compare two fractions with different numerators and different denominators by comparing to a benchmark fraction such as 1/2.
- Recognize that comparisons are valid only when two fractions refer to the same whole.
- Record the results of comparisons with symbols > , < , or =.
- Justify the conclusions for example by using a visual fraction model, number line, etc.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Justify their conclusions and communicate them to others.
- Learn to determine how their argument applies and make justifications.

Use appropriate tools strategically.

- Consider the available tool when solving a mathematical problem.
- Make sound decisions about when each tool is helpful.
- Recognize that some tools have limitations.

Essential questions

- How can you show that two fractions are equivalent using a visual fraction model?
- What strategies can be used to generate equivalent fractions?
- How does a benchmark fraction, like 1/2, help you compare fractions?
- What is a common denominator?
- How can you compare two fractions with like numerators and unlike denominators?
- How can you compare two fractions with like denominators and unlike numerators?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions[3]	**4.NF**

[3] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Extend understanding of fraction equivalence and ordering.

4.NF.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to

identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

In grades 1 and 2, students used fraction language to describe partitions of shapes into equal shares. In grade 3, students understood a fraction as a quantity formed by one part when a whole is partitioned into equal parts. They understood a fraction as a number on the number line and represented fractions on a number line. Students explained equivalence of fractions if two fractions were the same size or at the same point on a number line. Students recognized and generated simple equivalent fractions and explained using a visual fraction model. They expressed whole numbers as fractions and compared two fractions with the same numerator or denominator.

Current Learning

In grade 4, students explain why a fraction is equivalent to another fraction by using visual fraction models with attention to how the size of the parts are different, even though the two fractions are the same size. Students also compare two fractions with different numerators and denominators by creating common denominators or by comparing to a benchmark fraction. They recognize that these comparisons are only valid when two fractions refer to the same whole.

Future Learning

In grade 5, students will multiply a fraction by another fraction. Students will also divide a fraction by a nonzero whole number for future understanding of divisions of fractions by a fraction.

Additional Findings

According to *Principles and Standards for School Mathematics*, students in grades 3–5…

> "should develop strategies for ordering and comparing fractions often using benchmarks such as 1/2 and 1." (p. 150)

> "should begin to understand that between any two fractions there is always another fraction." (p. 150)

According to *A Research Companion to Principles and Standards for School Mathematics,* "To understand fractions as based in reciprocal relationships of relative size draws heavily on relationships among measure, multiplication, and division." (p. 108)