Grade 4 Mathematics, Quarter 3, Unit 3.1

Addition and Subtraction of Fractions

Overview

Number of instructional days: 10 (1 day = 60 minutes)

Content to be learned

- Add and subtract fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.
- Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$ (e.g., 3/7 = 1/7 + 1/7 + 1/7).
- Understand addition and subtraction of fractions as joining and separating fraction parts referring to the same whole.
- Decompose a fraction into a sum of fractions with the same denominator in more than one way.
- Record the decomposition of fractions with an equation (e.g., 3/8 = 1/8 + 1/8 + 1/8).
- Justify decomposition by using a visual fraction model.
- Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities and their relationship to problem situations.
- Create a coherent representation of the problem at hand.
- Attend to the meaning of quantities (not just how to compute them) and know and flexibly use different properties of operations and objects.

Look for and make use of structure.

- Recognize patterns and structure in the place value system.
- Recognize patterns and structure in adding and subtracting multidigit whole numbers.

Essential questions

- What do two fractions need to have in common in order to add or subtract?
- Given a fraction, how many ways can you decompose it by representing it as an equation?
- What does it mean to partition a whole?
- In what ways can you solve word problems involving adding and subtracting fractions with like denominators?
- How is partitioning of a whole representative of a given fraction?
- How can you represent a given fraction by partitioning a whole?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions[3]	**4.NF**

[3] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In grade 3, development of an understanding of fractions as numbers was essential for future work with the number system. Students placed fractions on a number line diagram. They started to develop the idea of fractions more formally, building on the idea of partitioning a whole into equal parts. The whole can be a shape such as a circle or rectangle, a line segment, or any one finite entity susceptible to subdivision and measurement.

Current Learning

In grade 4, students apply and extend their understanding of the meanings and properties of addition and subtraction in whole numbers to extend addition and subtraction to fractions. Students calculate sums of fractions with different denominators where one denominator is a divisor of the other, so that only one denominator has to be changed. The meaning of addition is the same for both fractions and whole numbers, even though algorithms for calculating their sums can be different. Later in the year, students start to add mixed numbers with like denominators and convert improper fractions to mixed numbers.

Future Learning

In grade 5, students will add and subtract fractions with unlike denominators by replacing given fractions with equivalent fractions so that they end up with like denominators. They will solve word problems involving addition and subtraction of fractions using benchmark fractions and number sense to estimate mentally and assess the reasonableness of answers on a visual fraction model. For the majority of the year, understanding of fractions will apply to multiplication and division.

Additional Findings

According to *Principles and Standards for School Mathematics*, for students in grades 3–5, "Emphasis should not be on developing general procedures to solve all fraction problems. Rather students should generate solutions that use a variety of models or representations." (p. 155)

According to the *Progressions for the Common Core State Standards in Mathematics 3–5 Number and Operations—Fractions,* "The simple understanding of addition as putting together allows students to see in a new light the way fractions a re built from unit fractions. Armed with this insight students compose and decompose fractions with the same denominators." (p. 6)

Grade 4 Mathematics, Quarter 3, Unit 3.2

Understanding Mixed Numbers

Overview

Number of instructional days: 15 (1 day = 60 minutes)

Content to be learned

- Add and subtract mixed numbers with like denominators by replacing each mixed number with an equivalent fraction.
- Apply and extend previous understanding of multiplication to multiply a fraction by a whole number.
- Understand a fraction a/b as a multiple of $1/b$ by using a visual fraction model and recording the conclusion with an equation [e.g., $5/4 = 5 \times (1/4)$].
- Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as 6/5.
- Solve word problems involving multiplication of a fraction by a whole number.
- Use properties of operations and the relationship between addition and subtraction to add and subtract mixed numbers with like denominators.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities and their relationship to problem situations.
- Create a coherent representation of the problem at hand.
- Attend to the meaning of quantities (not just how to compute them) and know and flexibly use different methods.

Model with mathematics.

- Apply the math they know to solve everyday problems.

Essential questions

- How can you represent an improper fraction as an equation with both a fraction and a whole number?
- What would the result be of multiplying a fraction by a whole number?
- In what ways can you add and subtract mixed numbers with like denominators?
- How can you solve word problems involving multiplication of a fraction by a whole number?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions[3]	**4.NF**

[3] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*

b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a

school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In grade 3, development of an understanding of fractions as numbers was essential for future work with the number system. Students placed fractions on a number line diagram. They started to develop the idea of fractions more formally by building on the idea of partitioning a whole into equal parts. The whole can be a shape such as a circle or rectangle, a line segment, or any one finite entity susceptible to subdivision and measurement.

Current Learning

In grade 4, students will extend previous understanding about how fractions are built from unit fractions, composing and decomposing fractions into unit fractions and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number. Students also compute sums of whole numbers and fractions (mixed numbers) by representing a whole number as an equivalent fraction with the same denominator as the fraction. They use this method to add and subtract numbers with like denominators.

Future Learning

In grade 5, students will use their understanding of fraction equivalence and their skill in generating equivalent fractions as a strategy to add and subtract fractions, including fractions with unlike denominators. They will solve word problems involving addition and subtraction of fractions using benchmark fractions and number sense to estimate mentally and assess the reasonableness of answers on a visual fraction model. For the majority of the year, understanding of fractions will apply to multiplication and division.

Additional Findings

According to *A Research Companion* to *Principles and Standards for School Mathematics*,

> "Students require a constant intertwining of understanding and doing—of building meaning, of problem solving, and of computing." (p. 91)
>
> "Ambitious mathematical goals; teacher-led and monitored discussion that focuses on central mathematical ideas; teachers' explaining and clarifying as well as children's explaining and clarifying; using and building on children's knowledge but extending that knowledge in mathematically important ways; and using carefully chosen real world context to facilitate all children's meaning-building." (p. 91)
>
> "Practice is important, as is learning prerequisite knowledge that facilitates acquiring more advanced methods." (p. 91)

Grade 4 Mathematics, Quarter 3, Unit 3.3

Understanding Decimals

Overview

Number of instructional days: 13 (1 day = 60 minutes)

Content to be learned

- Understand decimal notation for fractions and compare decimal notation.
- Express a fraction with denominator 10 as an equivalent fraction with denominator 100 and use this technique to add two fractions with respective denominators 10 and 100.
- Use decimal notation for fractions with denominators 10 or 100.
- Compare two decimals to hundredths by reasoning about their size.
- Recognize that comparisons are valid only when the two decimals refer to the same whole.
- Record the results of comparisons with the symbols >, =, or <.
- Justify your conclusions by using a visual model.

Mathematical practices to be integrated

Look for and make use of structure.

- Recognize patterns and structure in the place value system.
- Recognize patterns and structure in adding and subtracting multidigit whole numbers.

Look for and express regularity in repeated reasoning.

- Notice if calculations are repeated.
- Look for general methods and shortcuts.

Essential questions

- How can you compare fractions with a denominator of 10 and a denominator of 100?
- How can you represent fractions with a denominator of 10 or 100 as decimals?
- How can you compare two decimals to the hundredths by reasoning about their size?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions[3]	**4.NF**

[3] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Understand decimal notation for fractions, and compare decimal fractions.

4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[4] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*

[4] Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

4.NF.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Common Core Standards for Mathematical Practice

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In grade 3, students developed an understanding of fractions as numbers. They had experience with reasoning about the size of fractions using visual models and equivalence, but there was no decimal work.

Current Learning

In grade 4, students work with fractions with denominators 10 and 100, calling decimal fractions arising naturally when they convert between dollars and cents. They understand decimal notation for fractions and compare decimal fractions to the hundredths. Students compare decimals using the meaning of a decimal as a fraction, making sure to compare fractions with the same denominator. For example, to compare 0.2 and 0.09, students think of them as 0.20 and 0.09 and then see that 0.20 > 0.09 because 20/100 > 9/100. Students who can generate equivalent fractions develop general strategies for adding and subtracting, but addition and subtraction with unlike denominators is not a requirement.

Future Learning

In grade 5, students will recognize that in a multidigit number the value to the right is 1/10 of its place to the left. They will understand decimal notation for fractions and compare decimal fractions to the thousandths. Students will use place value to round decimals to any place. They will add, subtract, multiply, and divide decimals to the hundredths. Students will read, write, and compare decimals to the thousandths.

Additional Findings

According to *Principles and Standards for School Mathematics*,

> "An understanding of the base ten number system should be extended through continued work with larger numbers as well with decimals." (p. 149)
>
> "Students who understand the structure of numbers and the relationships among numbers can work with them flexibly." (p. 149)
>
> "The written place value system is a very efficient system that lets people write very large numbers. Yet it is very abstract and can be misleading: the digits in every place look the same. To understand the meaning of the digits in the various places, children need experience with some kind of *size/ quantity/supports*."