

Grade 4 Mathematics, Quarter 4, Unit 4.1
Representing and Interpreting Data

Overview

Number of instructional days: 10 (1 day = 60 minutes)

Content to be learned

- Make a line plot to display a data set of measurements and fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$).
- Analyze data on a line plot.
- Solve problems involving addition and subtraction of fractions by using information presented in line plots.

Essential questions

- How can you compare two data sets using information from a line plot?
- What does the X represent on a line plot?

Mathematical practices to be integrated

Model with mathematics.

- Represent problem situations in multiple ways.
- Connect representations and explain the connections.

Attend to precision.

- Communicate precisely to others.
- Specify units of measure carefully.
- Give carefully formulated explanations to each other.

- How can you determine the scale that is appropriate when making a line plot?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data

4.MD

Represent and interpret data.

- 4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 1, students organized, represented, and interpreted data with up to three categories. In grade 2, students showed measurements in a line plot where the horizontal scale is marked off in whole number units. Students also drew pictographs and bar graphs with single unit scales. In grade 3, students drew scaled pictographs and bar graphs with several categories and made line plots with a scale including whole numbers, halves, and quarters.

Current Learning

In grade 4, students make line plots with a scale in fractions of a unit, including $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and solve addition and subtraction problems based on their data involving the fractions of their scale.

Future Learning

In grade 5, students will continue to make line plots with a scale in fractions of a unit, including $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$. They will use operations on fractions that are part of their data.

Additional Findings

According to *Principles and Standards for School Mathematics*, for students in grades 3–5:

“Much of students’ work with data from grade 3–5 should involve comparing related data sets. Noting the similarities and differences between two data sets requires students to become more precise in their descriptions of the data.” (p. 179)

“Data can be used for developing arguments that are based on evidence and for continued problem posing.” (p. 180)

Grade 4 Mathematics, Quarter 4, Unit 4.2
Identifying Lines, Angles, and Shapes

Overview

Number of instructional days: 10 (1 day = 60 minutes)

Content to be learned

- Draw points, lines, line segments, rays, angles, and perpendicular and parallel lines and identify them in two-dimensional figures.
- Identify right, acute, and obtuse angles in two-dimensional figures.
- Recognize a triangle as a right triangle.
- Classify two-dimensional figures based on the presence or absence of these attributes.
- Recognize and draw a line of symmetry for a two-dimensional figure.

Essential questions

- What kind of attributes can be identified in two-dimensional figures?
- How can you determine if a shape has a line of symmetry?
- What is the difference between an acute and obtuse angle?

Mathematical practices to be integrated

Use appropriate tools strategically.

- Consider available tools, like a ruler, when solving a math problem.
- Students make sound decisions about when these tools are appropriate.

Attend to precision.

- Communicate precisely to others.
- Specifying units of measure carefully.
- Give carefully formulated explanations to each other.

- What specific math vocabulary can you use to classify two-dimensional figures.
- What is the difference between a parallel and perpendicular line?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry

4.G

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

- 4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
- 4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
- 4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Common Core Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 2, students learned to identify triangles, quadrilaterals, pentagons, and cubes. In grade 3, students understand that shapes in different categories may share the same attributes. These shared attributes define a large category (e.g., quadrilaterals). Students recognized that rhombuses, rectangles, and squares are examples of quadrilaterals. Students were able to draw examples of shapes that did not belong to any of these subcategories.

Current Learning

In grade 4, students draw and identify lines and angles, and classify shapes by properties of their lines and angles. These lines and angles include parallel and perpendicular lines, line segments, rays, right, acute and obtuse angles. Students recognize a line of symmetry as a line across a figure that divides it into matching parts. Students go from understanding these attributes to drawing them in this grade.

Future Learning

In grade 5, students will understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles. Students will classify two-dimensional figures in a hierarchy based on their properties.

Additional Findings

According to *Principles and Standards for School Mathematics*, students in grades 3–5:

“should develop more precise ways to describe shapes, focusing on identifying and describing the shapes properties and learning specialized vocabulary associated with these shapes and their properties.” (p. 165)

“should draw and construct shapes, compare and discuss their attributes, classify them, and develop and consider definitions on the basis of a shapes properties ...” (p. 165)

“should be expanding their mathematical vocabulary by hearing terms used repeatedly in context. As they describe shapes they should hear, understand, and use mathematical terms such as, parallel, perpendicular, angle, ... and so forth to communicate geometric ideas with greater precision.” (p. 166)

According to *A Research Companion to Principles and Standards for School Mathematics*, “Students believe that it is possible to abstract the properties of geometric objects from diagrams directly and thus deduce a property empirically.” (p. 159)

Grade 4 Mathematics, Quarter 4, Unit 4.3
Understanding and Measuring Angles

Overview

Number of instructional days: 10 (1 day = 60 minutes)

Content to be learned

- Measure angles in whole-number degrees using a protractor.
- Sketch angles of specified measures.
- Recognize angle measures as additive.
- Compose and decompose angles into nonoverlapping parts.
- Compose and decompose using addition and subtraction to find the unknown angles on a diagram.
- Recognize angles as geometric shapes where two rays share an endpoint.
- Recognize an angle is measured with reference to a circle.
- An angle has a measure of n degrees.

Essential questions

- How would you sketch and measure angles using a protractor?
- Given an equation with an unknown angle, how would you determine its measurement?

Mathematical practices to be integrated

Use appropriate tools strategically.

- Consider available tools like a ruler or protractor when solving a math problem.
- Make sound decisions about when these tools are appropriate.

Attend to precision.

- Communicate precisely to others.
- Specify units of measure carefully.
- Give carefully formulated explanations to each other.

- How is an arc of a circle related to the degrees of an angle?
- Why is knowing the measurement of angles important in real life?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data	4.MD
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Geometric measurement: understand concepts of angle and measure angles.

- 4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
- 4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.
- 4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:
- a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a “one-degree angle,” and can be used to measure angles.
 - b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.

Common Core Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 2, students learned to recognize and draw shapes with a given number of angles. The type of angle is not specified. Angles are only taught in the context of shapes in grade 3.

Current Learning

In grade 4, students understand concepts of angles and measure angles with reference to a circle and with a protractor. Students recognize angle measures as additive and are able to compose and decompose angles. This is the first time in which students are asked to find the actual degree measure of an angle.

Future Learning

In grade 5, students will recognize right, acute, and obtuse angles as attributes of a two-dimensional shape. The use of a mathematical tool like a protractor is not again used until grade 7.

Additional Findings

According to *Principles and Standards for School Mathematics*, students in grades 3–5:

“[when measuring] an object, the results should make sense; estimates and benchmarks can help students recognize when a measurement is reasonable.” (p. 243)

“should become proficient in composing and decomposing two-dimensional shapes. They should develop and understanding of different angle relationships and be proficient in measuring angles. ... they should learn to use a protractor to measure angles directly.” (p. 243)

“can begin to establish some benchmarks by which to estimate or judge the size of objects. For example they learn that a “square corner” is called a *right angle* and establish this as a benchmark for estimating the size of other angles.” (p.172)

Grade 4 Mathematics, Quarter 4, Unit 4.4
Problem Solving with Measurement

Overview

Number of instructional days: 10 (1 day = 60 minutes)

Content to be learned

- Use four operations to solve word problems involving distance, time, liquid volume, mass, and money.
- Solve problems involving simple fractions or decimals.
- Solve problems where a measurement given in a larger unit must be expressed in terms of a smaller unit.
- Represent measurement quantities using diagrams that feature a measurement scale. (e.g., a number line).

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Explain the meaning of a problem and look for ways to solve it.
- Check thinking by asking what makes sense and try different approaches.

Reason abstractly and quantitatively.

- Make sense of quantities and their relationship to problem situations.
- Create a coherent representation of the problem at hand.
- Attend to the meaning of quantities (not just how to compute them) and know and flexibly use different properties of operations and objects.

Essential questions

- What operations, materials or strategies can be used to solve problems involving: intervals of time, liquid volumes, mass, money, fractions, decimals, or distances?
- In what ways can you represent time, distance, volume, and mass using appropriate units of measure?
- How can you express measurements given in a larger unit in terms of a smaller unit?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data

4.MD

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Clarifying the Standards

Prior Learning

In grade 3, students told and wrote time to the nearest minute and solved word problems involving addition and subtraction of time intervals in minutes on a number line. They measured and estimated liquid volumes and masses of objects using grams, kilograms, and liters. They solved one-step word problems involving mass and volume in the same unit.

Current Learning

In grade 4, students use the four operations to solve word problems involving distance, time, volume, mass, and money with whole numbers, simple fractions, or decimals. Students expressed measurements given in a larger unit in terms of a smaller unit and used diagrams such as number lines.

Future Learning

In grade 5, students will convert among different sized standard measurements and solve multistep real-world problems within a given system. They will make line plots to display a data set of measurements in fractions of a unit and use operations on these fractions to solve problems involving the information on the line plot.

Additional Findings

According to *Principles and Standards for School Mathematics*, students in grades 3–5:

“in learning about measurements and learning how to measure, students should be actively involved drawing on familiar and accessible contexts.” (p. 171)

“should become familiar with the common units in the United States and metric systems and establish mental images or benchmarks for judging and comparing size.” (p. 172)

“should encounter the notion that measurements in the real world are approximate in part because of the instruments used and because of human error in reading the scales of these instruments.” (p. 172)

