Grade 5 Mathematics, Quarter 3, Unit 3.1

Dividing Fractions in the Real World

Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned

- Explain the division of a unit fraction (numerator is 1) by a nonzero whole number (greater than 0), and compute such quotients.
- Explain the division of a whole number by a unit fraction (numerator is 1), and compute such quotients.
- Solve real-world problems involving division of unit fractions (numerator is 1) by nonzero (greater than 0) whole numbers and division of whole numbers by unit fractions (numerator is 1).
- Interpret division problems using visual models.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships in problem situations.
- Pause as needed during the manipulation process to probe into the referents for the symbols involved.
- Consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

Model with mathematics.

- Look for and express regularity in repeated reasoning.
- Using tools such as diagrams, two-way tables, graphs, flowcharts, and formulas.

Essential questions

- What is an example of a real-world problem using division of fractions by whole numbers?
- What is an example of a real-world problem using division of whole numbers by fractions?
- How can you represent a division problem using fractions in a visual form? Show and explain your work.

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions	5.NF

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]

[1] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*

b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to

solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In grade 4, students learned the properties of equivalent fractions. They also compared, added, and subtracted fractions. Students added fractions with different denominators where one denominator is a divisor of the other, so that only one fraction has to be changed. They made connections with adding and multiplying fractions (e.g., $2/3 = 1/3 + 1/3 = 2 \times 1/3$). (*Progressions for the Common Core State Standards in Mathematics 3–5, Number and Operations—Fractions*, pp. 9–11)

Current Learning

In grade 5, students add and subtract fractions with unlike denominators using equivalent fractions to produce like denominators. They solve real-world problems using visual fraction models, equations, benchmark fractions, and number sense.

Students apply and extend their understanding by interpreting a fraction as division of the numerator by the denominator. They multiply a fraction or whole number by a fraction. Fifth graders find the area of a rectangle with fractional side lengths. They interpret multiplication as scaling. Students solve real-world problems with multiplication of fractions and mixed numbers. They divide fractions by whole numbers and whole numbers by fractions.

Students solve real-world problems using division of fractions by whole numbers and division of whole numbers by fractions.

Future Learning

In grade 6, students will solve word problems involving division of fraction by fractions, using visual fraction models and equations. Sixth graders will understand the concept of a ratio, associate a unit rate a/b with $a:b$ with $b \neq 0$, and use rate language. They will use rate and rate reasoning to solve real-world problems.

Additional Findings

In grade 5, students need not express the formula in general algebraic form, but rather reason out many examples using fraction strips and number line designs for more complicated examples an area model is useful. (*Progressions for the Common Core State Standards in Mathematics 3–5, Number and Operations—Fractions*, pp. 9–11)

Students work with multiplying by unit fractions and interpreting fractions in terms of division, enabling them to see that multiplying a quantity by a number smaller than 1 produces a smaller quantity.

Grade 5 Mathematics, Quarter 3, Unit 3.2

Understanding Volume

Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned

- Relate volume to multiplication and addition.
- Recognize volume as an attribute of solid figures.
- Recognize that a unit cube with a side length of one unit has one cubic unit of volume.
- Recognize that a unit cube can be used to measure volume.
- Recognize that the number of unit cubes used to pack a solid without gaps or overlaps is also the quantity of cubic units of volume of the solid.

Mathematical practices to be integrated

Model with mathematics.

- Identify important quantities in a practical situation and map their relationships.
- Use tools such as diagrams and formulas.

Attend to precision.

- Communicate precisely to others.
- Demonstrate care about specifying units of measure.

Look for and make use of structure.

- Recognize the significance of an existing line in a geometric figure.
- Look closely to discern a pattern or structure.

Essential questions

- What are the properties of a unit cube?
- How can a unit cube be used to measure volume?
- Why is it important to not leave spaces or gaps when measuring volume of a three-dimensional shape with unit cubes?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data	5.MD

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8

equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

Students used four operations to solve word problems involving liquid volumes. They combined their understanding of the meanings and properties of multiplication and division with their understanding of base-10 units to begin to multiply and divide multidigit numbers. In grade 4, students applied the area formulas for rectangles in real-world and mathematical problems.

Current Learning

Students fluently multiply multidigit whole numbers using the standard algorithm. Fifth graders extend their previous understanding of area to find the area of rectangle with fractional side lengths. Students recognize volume as an attribute of solid figures and understand concepts of volume measurement. They measure volumes by counting unit cubes. Students solve real-world and mathematical problems involving volume. They relate volume to packing with unit cubes to multiplying the edge lengths. Students apply the formulas $V = l \times w \times h$ and $V = b \times h$. They recognize volume as additive.

Future Learning

Sixth graders will solve real-world and mathematical problems involving area, surface area, and volume. They will continue to pack rectangular prisms with unit cubes and apply the formulas to find volume. Students will combine writing, reading, evaluating, and transforming variable expressions and solving equations and inequalities with the use of the volume formulas $V = l \times w \times h$ and $V = b \times h$.

Additional Findings

In grades 3–5, students should be able to recognize the need to select units appropriate to the attribute being measured. Likewise, the need for a standard three-dimensional unit to measure volume grows out of initial experiences filling containers with items such as rice or packing pieces. As students find that there are spaces between the units, the units are not easy to count, or units are not uniform, they will appreciate the need for a standard unit. (*Principles and Standards for School Mathematics*, p. 172)

Grade 5 Mathematics, Quarter 3, Unit 3.3

Measuring Volume Using Concrete Models

Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned

- Define volume as an attribute of solid figures.
- Measure volume by counting unit cubes, using cubic centimeter, cubic inch, and cubic feet units.
- Count unit cubes to measure volume using standard and improvised units.
- Fill rectangular prisms with counted unit cubes to show that the volume is the same as would be found by multiplying the edge lengths.
- Describe and express volume as a measurement using cubic units.
- Measure volume in cubic units by packing unit cubes in a solid figure.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Use the method of "accountable talk" to share information and build upon others' comments.
- Construct arguments using concrete referents such as objects, drawings, diagrams, and actions.
- Listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Model with mathematics.

- Apply the mathematics they know to solve problems in everyday life, society, and the workplace
- Identify important quantities in a practical situation and map their relationships using tools such as diagrams, two-way tables, graphs, flowcharts, and formulas.

Attend to precision.

- Use clear definitions in discussion with others and in own reasoning.
- Demonstrate care about using specific units of measure.
- Calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context.

Look for and express regularity in repeated reasoning

- Notice if calculations are repeated and look for general methods and shortcuts.
- Continually evaluate the reasonableness of intermediate results.

Essential questions

- How are standard units of measurement used when calculating volume with concrete models?
- How can you find the volume of a right rectangular prism using objects?
- Why do you express volume in "cubic units"?
- How can you use multiple arrays to show volume?
- How is volume an attribute of a solid figure?
- What is "one cubic unit" of volume?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data	**5.MD**

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

Students used four operations to solve word problems involving liquid volumes. They combined their understanding of the meanings and properties of multiplication and division with their understanding of base-10 units to begin to multiply and divide multidigit numbers. In grade 4, students applied the area formulas for rectangles in real-world and mathematical problems.

Current Learning

Students fluently multiply multidigit whole numbers using the standard algorithm. Fifth graders extend their previous understanding of area to find the area of rectangle with fractional side lengths. Students recognize volume as an attribute of solid figures and understand concepts of volume measurement. They measure volumes by counting unit cubes. Students solve real-world and mathematical problems involving volume. They relate volume to packing with unit cubes to multiplying the edge lengths. Students apply the formulas $V = l \times w \times h$ and $V = b \times h$. They recognize volume as additive.

Future Learning

Sixth graders will solve real-world and mathematical problems involving area, surface area, and volume. They will continue to pack rectangular prisms with unit cubes and apply the formulas to find volume. Students will combine writing, reading, evaluating, and transforming variable expressions and solving equations and inequalities with the use of the volume formulas $V = l \times \mathrm{w} \times h$ and $V = b \times h$.

Additional Findings

"Students should measure various rectangular solids using objects such as tiles and cubes, organize the information, look for patterns, and then make generalizations … These concrete experiences are essential in helping students understand the relationship between the measurement of an object and the succinct formula that produces the measurement." (*Principles and Standards for School Mathematics*, p. 175)

Grade 5 Mathematics, Quarter 3, Unit 3.4

Measuring Volume Using Formulas

Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned

- Find the volume of a right rectangular prism by packing it with unit cubes and linking this to multiplying the dimensions.
- Make connections between the formulas $V = l \times w \times h$ and $V = b \times h$ for finding the volume of rectangular prisms and the models for finding volume.
- Relate volume to multiplication and addition.
- Calculate volumes of solid figures composed of two nonoverlapping right rectangular prisms.
- Solve real-world problems involving volume.
- Apply the formulas for finding volume through application problems.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships in problem situations.
- Pause as needed during the manipulation process to probe into the referents for the symbols involved.
- Consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

Model with mathematics.

- Apply the mathematics they know to solve problems in everyday life, society, and the workplace
- Identify important quantities in a practical situation and map their relationships using tools such as diagrams, two-way tables, graphs, flowcharts, and formulas.

Look for and make use of structure.

- Look closely to discern a pattern or structure.
- Recognize the significance of an existing line in a geometric figure and use the strategy of drawing a secondary line for solving problems.

Essential questions

- How can you demonstrate the relationship between finding volume using unit cubes and multiplying the dimensions?
- Why is volume a three-dimensional measure?
- Why is volume measured in cubic units?
- How do you calculate volumes of solid figures composed of two nonoverlapping right rectangular prisms?
- What is the relationship between $V = b \times h$ and $V = l \times w \times h$?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data	5.MD

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Number and Operations in Base Ten	5.NBT

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships

mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

Students used four operations to solve word problems involving liquid volumes. They combined their understanding of the meanings and properties of multiplication and division with their understanding of base-10 units to begin to multiply and divide multidigit numbers. In fourth grade, students applied the area formulas for rectangles in real-world and mathematical problems.

Current Learning

Students fluently multiply multidigit whole numbers using the standard algorithm. Fifth graders extend their previous understanding of area to find the area of rectangle with fractional side lengths. Students recognize volume as an attribute of solid figures and understand concepts of volume measurement. They measure volumes by counting unit cubes. Students solve real-world and mathematical problems involving volume. They relate volume to packing with unit cubes to multiplying the edge lengths. Students apply the formulas $V = l \times w \times h$ and $V = b \times h$. They recognize volume as additive.

Future Learning

Sixth graders will solve real-world and mathematical problems involving area, surface area, and volume. They will continue to pack rectangular prisms with unit cubes and apply the formulas to find volume. Students will combine writing, reading, evaluating, and transforming variable expressions and solving equations and inequalities with the use of the volume formulas $V = l \times w \times h$ and $V = b \times h$.

Additional Findings

"Students should measure various rectangular solids using objects such as tiles and cubes, organize the information, look for patterns, and then make generalizations … These concrete experiences are essential in helping students understand the relationship between the measurement of an object and the succinct formula that produces the measurement." (*Principles and Standards for School Mathematics*, p. 175)