Grade 5 Mathematics, Quarter 4, Unit 4.1

Representing and Interpreting Data Using Fractions

Overview

Number of instructional days: 5 (1 day = 45 minutes)

Content to be learned

- Use line plots to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8).
- Solve problems using operations on fractions involving fractional measurement data displayed in the line plot.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Explain to themselves the meaning of a problem and look for entry points to its solution.
- Analyze givens, constraints, relationships, and goals.
- Can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends.

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships in problem situations.
- Create a coherent representation of the problem.
- Consider the units involved.
- Attend to the meaning of quantities, not just how to compute them, and know and flexibly use different properties of operations and objects.

Attend to precision.

- Communicate precisely to others.
- Carefully specify units of measure, labeling axes to clarify the correspondence with quantities in a problem.
- Calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context.

Essential questions

- How do you use the data on a line plot to perform operations involving fractions?
- How do you create a line plot with unit fractions as the scale?
- What are some questions that can be answered using your data?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data	**5.MD**

Represent and interpret data.

5.MD.2 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of

quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

Students made line plots to display data in fractions of a unit (1/2, 1/4, 1/8). They solved problems involving addition and subtraction of fractions by using information presented in line plots.

Current Learning

Students have frequent experiences with problems that are interesting, challenging, and engaging. They generate, organize, and evaluate information and results. Perseverance is an important aspect of the problem-solving process. Students understand the line plot representation of data, and with continual usage, they are able to interpret data.

Future Learning

In grade 6, measurement and data will extend to statistics and probability. Students will begin to work with rational numbers, including positive and negative numbers.

Additional Findings

According to *Principles and Standards for School Mathematics*,

> "Data can be used for developing arguments that are based on evidence and for continued problem posing. As students discuss data gathered to address a particular question, they should begin to distinguish between what the data show and what might account for the results." (p. 180)

> "In grades 3–5, students should have learned to generate and recognize equivalent forms of fractions, decimals, and percent, at least in some simple cases. In the middle grades, students should build on and extend this experience to become facile in using fractions, decimals, and percent meaningfully. Students can develop a deep understanding of rational numbers through experiences with a variety of models, such as fraction strips, number lines, 10×10 grids, area models, and objects. These models offer students concrete representations of abstract ideas and support students' meaningful use of representations and their flexible movement among them to solve problems." (p. 216)

Grade 5 Mathematics, Quarter 4, Unit 4.2

Geometry—Two-Dimensional Shapes

Overview

Number of instructional days: 5 (1 day = 45 minutes)

Content to be learned

- Recognize that attributes belonging to one category of two-dimensional shapes can also belong to the subcategories of two-dimensional shapes.
- Classify two-dimensional figures in a systematic order based on properties.

Mathematical practices to be integrated

Model with mathematics.

- Apply what they know to make assumptions and approximations.
- Analyze relationships mathematically to draw conclusions.

Attend to precision.

- Communicate precisely to others.
- Use clear definitions in discussion with others and in their own reasoning.
- Give carefully formulated explanations to peers.

Look for and make use of structure.

- Look closely to discern a pattern or structure.

Essential questions

- How can you use the attributes of two-dimensional figures to sort and classify?
- How can the attributes of a two-dimensional shape belong to a subcategory of another two-dimensional shape?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry	**5.G**

Classify two-dimensional figures into categories based on their properties.

5.G.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*

5.G.4 Classify two-dimensional figures in a hierarchy based on properties.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

Students recognized that area is an attribute of two-dimensional regions and extended their understanding of two-dimensional properties by finding area. They used transformations to design and analyze tiling and tessellations. They also measured and classified angles. Fourth graders drew points and identified lines, line segments, rays, angles, and perpendicular and parallel lines in two-dimensional figures. They recognized and drew lines of symmetry.

Current Learning

Fifth graders understand that attributes of one 2-dimensional figure belong to all subcategories as well. (For example, since rectangles have four right angles, they can classify two-dimensional figures in a systematic order based on properties.)

Future Learning

Sixth graders will find the area of triangles, quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes. They will apply these techniques to solve real-world and mathematical problems.

Additional Findings

According to *Principles and Standards for School Mathematics*, "As students investigate geometric properties and relationships their work can be closely connected with other mathematical topics, especially measurement and number … To consolidate their ideas, students should draw and construct shapes, compare and discuss their attributes, classify them and develop and consider definitions on the basis of a shape's properties." (p. 165)

Grade 5 Mathematics, Quarter 4, Unit 4.3

Geometry—Graphing/Coordinate Planes

Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned

- Understand that the intersection of the *x* and *y* axes is the origin point at (0, 0).
- Understand that the *x*-axis runs horizontally.
- Understand that the *y*-axis runs vertically.
- Distinguish between *x*-axis and *y*-axis.
- Understand that the value of *x* indicates the travel direction from the point of origin on the *x*-axis.
- Understand that the value of *y* indicates the travel direction from the point of origin on the *y*-axis.
- Understand that an ordered pair is always written as (*x*, *y*).
- Locate a point by using an ordered pair in Quadrant 1.
- Given a point(s) in Quadrant 1, provide the corresponding ordered pair.
- Graph and interpret points on a coordinate plan in real-world mathematical problems.

Mathematical practices to be integrated

Model with mathematics.

- Identify important quantities in a practical situation and map their relationships using such tools as graphs.

Use appropriate tools strategically.

- Consider the available tools when solving a mathematical problem.
- Be sufficiently familiar with tools appropriate for their grade or course to make sound decisions.
- Know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.

Attend to precision.

- Communicate precisely to others.
- Carefully specify and label axes to clarify the correspondence with quantities in a problem.

Essential questions

- What do the *x*- and *y*-axes on a coordinate plane represent?
- What is the connection between the *x*- and *y*-axes on a coordinate grid and an ordered pair?
- How would you graph an ordered pair on a coordinate plane?
- Using data from a table, how would you graph ordered pairs?
- How would you name a point in the first quadrant on the coordinate plane?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry	5.G

Graph points on the coordinate plane to solve real-world and mathematical problems.

5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).

5.G.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use

them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In earlier grades, students solved problems by making frequency tables, bar graphs, picture graphs, and line plots. They used stem-and-leaf plots.

Current Learning

Students build on their previous work with number lines to use two perpendicular number called axes to define a coordinate system. They understand how to plot ordered pairs and how to name points on the coordinate plane. They represent real-world and mathematical problems by graphing points.

Future Learning

In grade 6, students will extend the coordinate plane to represent rational numbers and ordered pairs. They will work with negative number coordinates. They will understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane. They will find and position pairs of integers and other rational numbers in a coordinate plane.

Additional Findings

According to *Principles and Standards for School Mathematics*, “Experiences with the rectangular coordinate plane will be useful as they solve a wider array of problems in geometry and algebra.” (p. 43)

Grade 5 Mathematics, Quarter 4, Unit 4.4

Equations and Graphing

Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned

- Generate two numerical patterns using two given rules.
- Identify relationships between corresponding terms.
- Form ordered pairs consisting of corresponding terms from the two patterns.
- Graph the ordered pairs on a coordinate plane.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Display the ability to decontextualize—to abstract a given situation, represent it symbolically, and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents.
- Display the ability to contextualize—to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.
- Create a coherent representation of the problem at hand.

Look for and make use of structure.

- Look closely to discern a pattern or structure.
- See complicated things such as some algebraic expressions as single objects or as being composed of several objects.

Essential questions

- How can you use two numerical patterns to generate ordered pairs and graph those ordered pairs in the first quadrant of the coordinate plane?
- How do you use a rule to generate ordered pairs?
- How do you determine a pattern from ordered pairs?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	5.OA

Analyze patterns and relationships.

5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

"Students worked with number lines. Students began to generate and analyze patterns." (*CCSS Progressions*, p. 22)

Current Learning

According to *CCSS Progressions*,

> "Students use perpendicular number lines to define a coordinate system. Students work with numerical patterns that can be related. They examine these relationships within sequences of ordered pairs and in the graphs in the first quadrant of the coordinate plane."
>
> "Students create graphs of simple equations. They construct and analyze double-bar and line graphs and use ordered pairs on coordinate graphs. Students begin working more formally with expressions. They write expressions to express a calculation [$2 \times (8 + 7)$ 'add 8 and 7 then multiply by 2']. They evaluate and interpret expressions [$3 \times (546 + 729)$ is three times as large as $546 + 729$] without having to calculate the answer."
>
> "Expressions should be found in the associative or distributive property $(8 + 27) + 2$ or $(6 \times 30) + (6 \times 7)$. Numbers need not always be whole numbers."

Future Learning

According to *CCSS Progressions*, "Students will work with variable expression (three times an unknown length is 3 • L). Students will study proportional relationships and functions. They will interpret the structure of an expression in terms of a context."

Additional Findings

According to *Principles and Standards for School Mathematics*, "Experiences with the rectangular coordinate plane will be useful as they solve a wider array of problems in geometry and algebra." (p. 43)

Grade 5 Mathematics, Quarter 4, Unit 4.5

Working with Numerical Expressions

Overview

Number of instructional days: 5 (1 day = 45 minutes)

Content to be learned

- Use parentheses, brackets, or braces in numerical expressions and evaluate expressions with these symbols.
- Write simple expressions to record calculations involving numbers only.
- Interpret numerical expressions without actually calculating them.

Mathematical practices to be integrated

Model with mathematics.

- Make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.
- Routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Attend to precision.

- Communicate precisely to others.
- State the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- Calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context.

Essential questions

- When would numbers need to be grouped together using parentheses, brackets, or braces?
- What is a real-life example of when it makes sense to group numbers together?
- When given a context or expression requiring more than one operation between two numbers, how do you make sense of which calculation to do first?
- When solving equations, what does the use of parentheses, brackets and braces tell you about the order in which you should work?
- Without performing any calculations, what can you tell me about the value of this expression? (e.g., The value of $2 \times (8 + 7)$ will be twice as big as the value $8 + 7$.)

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	5.OA

Write and interpret numerical expressions.

5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

As early as grade 3, students solved two-step problems. In doing so, they informally used the Associate and Distributive Properties [e.g., $(2 \times 3) \times 4$ versus $2 \times (3 \times 4)$ and 4×7 versus $4 \times (5 + 2)$]. These properties were the heart of grades 3–5 mathematics, from multiplication and division strategies involving whole numbers and decimals to all operations with fractions. Understanding these properties, even informally, required students to know to perform operations inside the grouping symbols first, and perform multiplication and division before addition and subtraction. In grades 3–5, parentheses were used for these situations, but fluency was not expected until grade 6.

Students in grade 3 knew how to perform operations in the conventional order where there are no parentheses to specify a particular order (orders of operation). Fourth graders continued to use properties of operations and should have known that they multiply and divide (from left to right) before they add and subtract (from left to right).

Current Learning

At this level, fifth graders evaluate expressions (not restricted to whole numbers) with parentheses ( ), brackets [ ], and braces { }. Grouping symbols can be thought of as hands curved around the symbols and grouping them together.

Students write simple expressions to express calculations. Expressions are a series of numbers and symbols (+, –, ×, /) without an equal sign. $6 \times (7.5 + 3.1)$ is an example of an expression. Students should be able to use their conceptual understanding of multiplication to interpret expressions without performing calculations. For example, interpret $6 \times (7.5 + 3.1)$ as being 6 times greater than the sum 7.5 + 3.1. This learning is not expected to be at the mastery level and therefore should be restricted to simple expressions involving the Associative and Distributive Properties—not parentheses nested within other grouping symbols. Note, however, the numbers are not restricted to whole numbers.

Future Learning

In grade 6, students will view expressions not just as calculations, but also as objects with structure (e.g., three times an unknown length is 3***l***). Sixth graders will write, read, and evaluate expressions in which letters stand for numbers. They will identify parts of an expression using mathematical terms (e.g., sum, term, factor) and will view one or more parts of an expression as a single entity. Students will evaluate expressions at specific values of their variables, including real-world problems. They will apply properties of operations to generate equivalent expressions and use the properties to identify when two expressions are equivalent.

Additional Findings

According to *Principles and Standards for School Mathematics*, "Algebra is more than moving symbols around. Students need to understand the concepts of algebra, the structures and principles that govern the manipulation of the symbols, and how the symbols themselves can be used for recording ideas and gaining insights into situations." (p. 37)